CANONICAL SCALE SEPARATION IN 2D INCOMPRESSIBLE HYDRODYNAMICS

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The fundamental rules governing a two-dimensional inviscid incompressible fluid are simple. Yet, to characterize the long-time behaviour is a knotty problem. The fluid's motion is described by Euler's equations: a non-linear Hamiltonian system with infinitely many conservation laws. In both experiments and numerical simulations, coherent vortex structures, or blobs, emerge after an initial stage. These formations dominate the large-scale dynamics, but small scales also persist. In his classical work, Kraichnan qualitatively describes a forward cascade of enstrophy into smaller scales and a backward cascade of energy into larger scales. Previous attempts to model Kraichnan's double cascade use filtering techniques that enforce separation from the outset.

In this talk, we show that Euler's equations posses an intrinsic, canonical splitting of the vorticity function. The splitting is remarkable in four ways:

- it is defined solely via the Poisson bracket and the Hamiltonian,
- it characterizes steady flows,
- without imposition it yields a separation of scales, enabling the dynamics behind Kraichnan's qualitative description,
- it accounts for the "broken line" in the power law for the energy spectrum (observed in both experiments and numerical simulations).

The splitting originates from Zeitlin's truncated model of Euler's equations in combination with a standard quantum-tool: the spectral decomposition of Hermitian matrices. In addition to theoretical insight, the scale separation dynamics could be used for stochastic model reduction, where small scales are modelled by a suitable multiplicative noise.

This is a joint work with prof. Klas Modin.

References

[1] K. Modin and M. Viviani, *Canonical scale separation in two-dimensional incompressible hydrodynamics*, arXiv, 2021

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