A TOPOLOGICAL ANALYSIS OF COMPETITIVE ECONOMIES

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Consider a multitude of decision makers whose choices are affected by the totality of others' actions, and yet negligible when considered individually. Whether agents act independently or coordinate their moves, the global outcome of their interaction is driven only by the aggregate result of their decisions. This situation is typical of competitive economies, where the aggregation process associates each group of agents with a set of bundles representing their mean demands.

In this presentation we ask what conditions ensure that aggregating many agents has a convexifying effect on their mean demands. The problem is an old one and characterizes competitive economies. We focus on the case of economies with an infinite-dimensional commodity space, where the classical results based on Aumanns' non-atomic representation of agents do not apply.

The question translates in the following mathematical problem: let $\mathcal{R}$ be a Boolean ring, E a locally convex linear space and $\varphi\colon \mathcal{R} \to 2^E$ a correspondence that is additive in the sense that $\varphi(0) = \{0\}$ and $\varphi(a \vee b) = \varphi(a) + \varphi(b)$ whenever a and b are disjoint. Under what conditions is the range $\bigcup\{\varphi(a) : a \in \mathcal{R}\}$ a convex and weakly compact set?

Our main results gives conditions under which an additive correspondence has a convex and weakly compact range. We base our approach on a topological reformulation of the "saturation property", a condition introduced in [1] and recently employed to prove specific extensions of Lyapunov's Theorem on the range of vector measures (see [2] and its references).

When included in the economic model, our conditions provide further insights on the nature of competitive economies in an infinite-dimensional context. We use our results to extend classical properties on the veto-power of small coalitions in the spirit of [3] and [4].

References

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