BILINEAR CONTROL OF EVOLUTION EQUATIONS ON COMPACT NETWORKS

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In the recent work [1] we have solved the problem of exact controllabilty, in finite time $T > 0$, of the parabolic evolution problem

$$\begin{cases} u'(t) + Au(t) + p(t)Bu(t) = 0 \\ u(0) = u_0 \end{cases} \tag{1}$$

to some special target trajectories, called *eigensolutions*. Here p is a bilinear control. Denoted by $\{\lambda_k\}_{k\in\mathbb{N}^*}$ the eigenvalues of A and by $\{\varphi_k\}_{k\in\mathbb{N}^*}$ the associated eigenfunctions, the jth eigensolution of (1), $\psi_j(t) = e^{-\lambda_j t}\varphi_j$, is the solution of (1) for $p = 0$ and $u_0 = \varphi_j$.

The hypotheses to apply our result are linked to the null controllability of the following linearized problem

$$\begin{cases} u'(t) + Au(t) + p(t)B\varphi_j = 0 \\ u(0) = u_0 \end{cases}$$

and to the associated control cost. Sufficient conditions to have a suitable control cost, which allows to apply our controllability result, are an uniform gap condition of the eigenvalues of A and a lower bound for the Fourier coefficients of $B\varphi_j$. Observe that, because of the gap condition, the results of [1] are mostly applicable to low dimensional problems.

Therefore, it is reasonable to apply our controllability result to parabolic evolution equation on network structure, which are essentially one-dimensional domains. However, by considering the following dynamics

$$\begin{cases} u_t(t,x) - \Delta u(t,x) + p(t)Bu(t,x) = 0 \\ u(0,x) = u_0(x) \end{cases}$$

on a graph, one soon realizes that the eigenvalues of the Laplacian do not verify an uniform gap.

In [2] we adapted the controllabilty result of [1] to the case of a weaker gap condition of the eigenvalues. Thus, we were able to prove controllability to eigensolutions for the above problem on star and tadpole graphs.

This is a joint work with P. Cannarsa and A. Duca.

References

[1] F. Alabau-Boussouira, P. Cannarsa and C. Urbani, *Exact controllability to eigensolutions for evolution equations of parabolic type via bilinear control*, to appear in NoDEA, preprint available on arXiv:1811.08806, 2022

[2] P. Cannarsa, A. Duca and C. Urbani, *Exact controllability to eigensolutions of the bilinear heat equation on compact networks*, Discrete and Continuous Dynamical Systems - Series S, 2022

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