In the recent work [1] we have solved the problem of exact controllability, in finite time $T > 0$, of the parabolic evolution problem

\begin{equation}
\begin{cases}
u'(t) + Au(t) + p(t)Bu(t) = 0 \\
u(0) = u_0
\end{cases}
\end{equation}

to some special target trajectories, called eigensolutions. Here $p$ is a bilinear control. Denoted by $\{\lambda_k\}_{k \in \mathbb{N}^*}$ the eigenvalues of $A$ and by $\{\varphi_k\}_{k \in \mathbb{N}^*}$ the associated eigenfunctions, the $j$th eigensolution of (1), $\psi_j(t) = e^{-\lambda_j t}\varphi_j$, is the solution of (1) for $p = 0$ and $u_0 = \varphi_j$.

The hypotheses to apply our result are linked to the null controllability of the following linearized problem

\begin{equation}
\begin{cases}
u'(t) + Au(t) + p(t)B\varphi_j = 0 \\
u(0) = u_0
\end{cases}
\end{equation}

and to the associated control cost. Sufficient conditions to have a suitable control cost, which allows to apply our controllability result, are an uniform gap condition of the eigenvalues of $A$ and a lower bound for the Fourier coefficients of $B\varphi_j$. Observe that, because of the gap condition, the results of [1] are mostly applicable to low dimensional problems.

Therefore, it is reasonable to apply our controllability result to parabolic evolution equation on network structure, which are essentially one-dimensional domains. However, by considering the following dynamics

\begin{equation}
\begin{cases}
u_t(t, x) - \Delta u(t, x) + p(t)Bu(t, x) = 0 \\
u(0, x) = u_0(x)
\end{cases}
\end{equation}

on a graph, one soon realizes that the eigenvalues of the Laplacian do not verify an uniform gap.

In [2] we adapted the controllability result of [1] to the case of a weaker gap condition of the eigenvalues. Thus, we were able to prove controllability to eigensolutions for the above problem on star and tadpole graphs.

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**References**
