A PULLBACK FUNCTOR FOR L^2-COHOMOLOGY

STEFANO SPESSATO

The main argument of this seminar is a result of the author in L^2-cohomology ([2], [3]). The definition of L^2-cohomology is very similar to the definition of de Rham cohomology. Indeed, given an oriented, possibly not compact, complete Riemannian manifold (M,g), the k-th L^2-cohomology group of (M,g) is

$$H_2^k(M,g) := \frac{ker(d^k)}{im(d^{k-1})}. \tag{1}$$

The operator $d^k : \Omega_2^k(M,g) \longrightarrow \Omega_2^{k+1}(M,g)$ is the exterior derivative operator of smooth forms defined on $\Omega_2^k(M,g)$, which is the space of smooth forms ω such that both ω and $d\omega$ are square integrable.

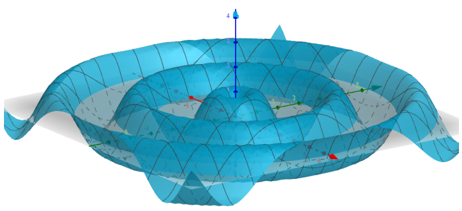
Differently from the de Rham cohomology, the pullback f^* along a map f is not well-defined as operator between $\Omega_2^*(N,h)$ and $\Omega_2^*(M,g)$. Several examples from [2] and [3] will be showed. During the seminar we will see how, fixed a map $f : (M,g) \longrightarrow (N,h)$, is possible to define a bounded operator $T_f : \Omega_2^*(N,h) \longrightarrow \Omega_2^*(M,g)$. The definition of T_f is inspired by a similar operator for compact manifolds defined by Hilsum and Skandalis in [1].

In order to define T_f we need two assumptions. The first one is about the Riemannian metrics: (M,g) and (N,h) have to be manifolds *of bounded geometry*, which means that there are an upper bound on the norm of the curvature and a lower bound on the injectivity radius. The second assumption is about the map f. We require that f is a *uniform* map, i.e. f is uniformly continuous and for each subset A of N, the diameter of $f^{-1}(A)$ is bounded in terms of the diameter of A itself.

As a consequence we obtain a contravariant functor

$$\begin{cases} \mathcal{T}(M,g) = H_2^k(M,g) \\ \mathcal{T}(f : (M,g) \longrightarrow (N,h)) = f^\circ : H_2^k(N,h) \longrightarrow H_2^k(M,g) \end{cases} \tag{2}$$

where $f^\circ[\omega] := [T_f\omega]$. Finally we obtain that the L^2-cohomology is a uniform homotopy invariant for manifolds of bounded geometry. Then, time permitting, we will see some consequences of this invariance.



References

[1] M. Hilsum, G. Skandalis; *Invariance par homotopie de la signature a coefficients dans un fibre presque plat*, J. für die reine und angewandte Mathematik; Jan 1992;

[2] S.Spessato; *A pullback functor for (un)-reduced $L^{q,p}$-cohomology*, preprint: https://arxiv.org/pdf/2102.08653.pdf;

[3] S.Spessato; *Lipschitz-homotopy invariants: L^2-cohomology, Roe index and ρ-class*, Ph.D. Thesis: https://arxiv.org/pdf/2109.07400.pdf.

Università degli Studi Niccolò Cusano
Email address: stefano.spessato@unicusano.it