A GEOMETRIC SINGULAR PERTURBATION APPROACH TO EPIDEMIC COMPARTMENTAL MODELS

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We study fast-slow versions of the SIR, SIRS and SIRWS epidemiological models [1], and of the SIRS epidemiological model on homogeneous graphs [2], obtained through the application of the moment closure method. The multiple time scale behavior is introduced to account for large differences between some of the rates of the epidemiological pathways.

Our main purpose is to show that the fast-slow models, even though in nonstandard form, can be studied by means of Geometric Singular Perturbation Theory (GSPT).

In particular, without using Lyapunov's method, we are able to not only analyze the stability of the endemic equilibria of the SIR and SIRS models, but also to show that in the remaining models limit cycles arise.

We show that the proposed approach is particularly useful in more complicated (higher dimensional) models such as the SIRWS model and the SIRS on homogeneous graphs, for which we provide a detailed description of their dynamics by combining analytic and numerical techniques. In particular, for the latter we show that the model can give rise to periodic solutions, differently from the corresponding model based on homogeneous mixing.

References

[2] JARDÓN-KOJAKHMETOV, H., KUEHN, C., PUGLIESE, A. AND SENSI, M., A geometric analysis of the SIRS epidemiological model on a homogeneous network, Journal of Mathematical Biology, 83.4 (2021): 1-38.

^[1] JARDÓN-KOJAKHMETOV, H., KUEHN, C., PUGLIESE, A. AND SENSI, M., A geometric analysis of the SIR, SIRS and SIRWS epidemiological models, Nonlinear Analysis: Real World Applications, 58 (2021): 103220.