GEOMETRIC MEASURE THEORY UNDER LOWER RICCI CURVATURE BOUNDS: A NON SMOOTH PERSPECTIVE

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In Geometric Analysis there is a celebrated connection between minimal hypersurfaces and Ricci curvature. This interplay essentially boils down to the presence of Ricci in the second variation formula for the area and it is source of difficulties when the minimal objects obtained through Geometric Measure Theory (currents, varifolds, sets of finite perimeter) are not smooth, in high dimension.

The striking growth of the synthetic theory of lower Ricci curvature bounds in recent years led to a natural question: how many of the classical results linking Geometric Measure Theory with Ricci curvature hold when also the ambient space is not smooth?

In this talk I will review some of the most recent developments in this setting and illustrate how they can be employed to solve classical open problems in Geometric Analysis.

The talk is based on joint works in collaboration with Luigi Ambrosio, Gioacchino Antonelli, Elia Bruè, Andrea Mondino, Enrico Pasqualetto and Marco Pozzetta.

References

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