FANO MANIFOLDS WITH LEFSCHETZ DEFECT 3

SAVERIO ANDREA SECCI

The *Lefschetz defect* δ_X is a numerical invariant associated to a smooth complex, Fano variety X, and it depends on the Picard number of prime divisors contained in X. Explicitly, consider the real vector space $\mathcal{N}_1(X)$ of real 1-cycles up to numerical equivalence. Its dimension is the Picard number ρ_X. Now, for any prime divisor D in X we define $\mathcal{N}_1(D,X)$ as the image of the pushforward $\iota_*\colon \mathcal{N}_1(D) \to \mathcal{N}_1(X)$ induced by the inclusion $\iota\colon D \hookrightarrow X$, that is the subvector space of $\mathcal{N}_1(X)$ generated by the numerical classes in X of curves in D. Finally,

$$\delta_X := \max\{\operatorname{codim}\mathcal{N}_1(D,X) | D \text{ a prime divisor in } X\}.$$

The main property of δ_X is that if $\delta_X \geq 4$, then X is isomorphic to a product $S \times T$, with S a del Pezzo surface of Picard number $\rho_S = \delta_X + 1$ [1, Th. 3.3].

In this talk we discuss a classification result for smooth Fano varieties with $\delta_X = 3$, which provides a generalisation of [2] to any dimension and Picard number: although X is not necessarily a product, it still has a very explicit description. That is, there exist a smooth Fano variety T of dimension $\dim T = \dim X - 2$ and Picard number $\rho_T = \rho_X - 4$, and a fibration $\sigma\colon X \to T$ such that the fibres are del Pezzo surfaces, and σ factorises as a $\mathbb{P}^2$-bundle over T and the blow-up along three pairwise disjoint smooth, irreducible, codimension 2 subvarieties, horizontal for the $\mathbb{P}^2$-bundle over T. We explicitly describe all possible $\mathbb{P}^2$-bundles and centres of the blow-up.

Moreover, we see some applications of the structure theorem: we describe the fibres, the relative contractions and the different factorisations of σ, and finally we describe in more details the case $\rho_X = 5$ and conclude the classification for $\dim X = 4$.

This talk is based on a joint work with C. Casagrande and E. A. Romano [3].

References

[1] C. Casagrande, *On the Picard number of divisors in Fano manifolds*, Ann. Sci. Éc. Norm. Supér. Volume 45, pp. 363–403 (2012).

[2] C. Casagrande, E. A. Romano, *Classification of Fano 4-folds with Lefschetz defect 3 and Picard number*, J. Pure Appl. Algebra, Volume 226, Issue 3 (2022).

[3] C. Casagrande, E. A. Romano, S. A. Secci, *Fano manifolds with Lefschetz defect 3*. Preprint arXiv:2201.02413 (2022).

University of Turin – Dep. of Mathematics 'G. Peano'
Email address: saverioandrea.secci@unito.it