AUTOMORPHISMS ON ALGEBRAIC VARIETIES: K3 SURFACES, HYPERKÄHLER MANIFOLDS, AND APPLICATIONS ON ULRICH BUNDLES

YULIETH PRIETO–MONTANEZ

One of the main tools to study the geometry of complex algebraic varieties is the group of automorphisms. The goal of this talk is first of all to show how a non–empty automorphism group can determine naturally the geometry of some hyperkähler manifolds *or* can characterize varieties admitting objects (e.g., vector bundles) with specific conditions. It is not our purpose to determine the group of automorphisms of some algebraic varieties, instead, we use the action of (birational maps) automorphisms in different contexts, to restrict the geometry of the variety or in some cases the existence of certain objects.

As time is pressing, I will present only some ideas of the following results:

Theorem 1. *Let X be a smooth projective variety with non–empty $Bir(X)$.*

(1) *If X is a K3 surface admitting a symplectic automorphism σ of order three, then $\rho(X) \geq 13$ and if $\rho(X) = 13$ the Nerón–Severi group of X determines uniquely the K3 surface of the minimal resolution of $X/\langle\sigma\rangle$ and viceversa.*

(2) *If X is a hyperkähler manifold of $K3^{[n]}$–type admitting a symplectic birational map with non–trivial action on the discriminant group of $H^2(X,\mathbb{Z})$, then X is birational to a moduli space of (twisted) sheaves on a K3 surface.*

(3) *If the tangent bundle of X is an Ulrich bundle, the X is a rational homogeneous space with a rather large automorphism group. Moreover, if $\dim X = 1$, then X is the twisted cubic in $\mathbb{P}^3$ and if $\dim X = 2$, then X is the Veronese surface surface in $\mathbb{P}^5$.*

These are the main results on my PhD thesis [1]. Part (1) is a joint work with A. Garbagnati in [2]. Part (3) is a joint work with P. Montero, S. Troncoso, and V. Benedetti in [3].

References

[1] Y. Prieto–Montañez, *Automorphisms on algebraic varieties: K3 surfaces, hyperkähler manifolds, and applications on Ulrich bundles*, 2022.

[2] A. Garbagnati,Y. Prieto–Montañez, *Order 3 symplectic automorphisms on K3 surfaces*, Mathematische Zeitschrift, 2021.

[3] V. Benedetti, P. Montero, Y. Prieto, S. Troncoso, *Projective manifolds whose tangent bundle is Ulrich*, *arXiv:2108.13944v2*, 2021.

Università di Bologna/ Riemann Center for Geometry and Physics
Email address: yulieth.prieto2@unibo.it
URL: https://sites.google.com/view/yuliethprietom/home