SMOOTHING EFFECTS AND INFINITE TIME BLOWUP FOR REACTION-DIFFUSION EQUATIONS: AN APPROACH VIA SOBOLEV AND POINCARÉ INEQUALITIES

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We consider the following Cauchy problem for a class of reaction-diffusion equations

$$
\begin{cases} u_t = \Delta u^m + u^p & \text{in } M \times (0,T) \\ u = u_0 & \text{in } M \times \{0\} \end{cases} \tag{1}
$$

where M be a complete noncompact Riemannian manifold of infinite volume. Here $m > 1$, which is known as slow diffusion of the porous medium type. We consider the particularly delicate case $1 < p < m$ in problem (1), a case largely left open in [1] even when the initial datum is smooth and compactly supported. We prove global existence for L^m data, and a smoothing effect for the evolution, i.e. that solutions corresponding to such data are bounded at all positive times with a quantitative bound on their L^∞ norm. As a consequence of this fact and of a result of [1], it follows that on Cartan-Hadamard manifolds with curvature pinched between two strictly negative constants, solutions corresponding to sufficiently large L^m data give rise to solutions that blow up pointwise everywhere in infinite time, a fact that has no Euclidean analogue. The methods of proof of the smoothing effect are functional analytic in character, as they depend solely on the validity of the Sobolev inequality and on the fact that the L^2 spectrum of Δ on M is bounded away from zero (namely on the validity of a Poincaré inequality on M).

This is a joint work with Gabriele Grillo and Fabio Punzo.

References

[1] G. Grillo, M. Muratori, F. Punzo, *B*low-up and global existence for the porous medium equation with reaction on a class of Cartan-Hadamard manifolds, J. Diff. Eq. **266** (2019), 4305-4336.

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