NONLOCAL MINIMAL GRAPHS

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The fractional perimeter and the associated minimizing sets, whose boundaries are the socalled nonlocal minimal surfaces, were introduced by Caffarelli, Roquejoffre and Savin in [1], in 2010, and have since then attracted a lot of interest.

In this talk I will present some recent results, that I have obtained with Matteo Cozzi, in [2], concerning subgraphs that have finite fractional perimeter. A preliminary version of [2] can be found in my PhD thesis [3].

We will define a fractional and nonlocal counterpart of the classical area functional, then we will focus on studying its minimizers, the nonlocal minimal graphs. In particular, I will prove the existence and uniqueness of minimizers, under very mild assumptions on the prescribed exterior datum. By exploiting a vertical rearrangement inequality, I will then show that the subgraphs of the minimizers of the fractional area functional minimize the fractional perimeter. Finally, I will give a sketch of the proof of the equivalence of nonlocal minimal graphs and solutions of the fractional mean curvature equation.

References

- L. CAFFARELLI, J.-M. ROQUEJOFFRE & O. SAVIN, Nonlocal minimal surfaces, Comm. Pure Appl. Math. 63, no. 9, 1111–1144, 2010.
- M. COZZI & L. LOMBARDINI, On nonlocal minimal graphs, Calc. Var. Partial Differential Equations 60, no. 4, Paper No. 136, 72 pp., 2021.
- [3] L. LOMBARDINI, Minimization problems involving nonlocal functionals: nonlocal minimal surfaces and a free boundary problem, PhD thesis, available at arXiv:1811.09746, 2018.

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