Atiyah, in [1], using \(\zeta\)-regularization techniques and the equivariant localization theorem, recovered the \(\hat{A}\)-genus of a Spin manifold as a class in equivariant cohomology. In particular he showed that (roughly)

\[
\hat{A}(M) = \int_M \frac{1}{\text{eul}(\nu)}
\]

where \(\nu\) is the (infinite rank, \(S^1\)-equivariant) normal bundle of \(\iota : M \to \mathcal{L}M = \text{Maps}(S^1, M)\) and \(\text{eul}(\nu)\) is its Euler class.

In [2], we generalize this fact to the space of maps from elliptic curves and to the Witten genus. In particular, introducing a suitable “anti-holomorphic sector” we prove a generalization of the equivariant localization theorem. This allows us to consider smooth actions of elliptic curves \(\mathbb{C}/\Lambda\) and thus lets us apply \(\zeta\)-regularization techniques, with rigour not yet found in the literature, to recover the Witten genus as the modular form given (again, roughly), over a point \(\tau \in \mathbb{H}\), by the formula

\[
\text{Wit}(M)(\tau) = \int_M \frac{1}{\text{eul}(\nu_\tau)}
\]

where \(\nu_\tau\) is the (infinite rank, \(\mathbb{C}/\Lambda\)-equivariant) normal bundle of \(\iota : M \to \text{Maps}(\mathbb{C}/\Lambda, M)\), with \(\Lambda\) the lattice generated by 1 and \(\tau\), \(M\) a rational String manifold, and \(\text{eul}(\nu)\) the Euler class of \(\nu\).

After a brief introduction of the localization theorem in both the classical and antiholomorphic setting, I will survey some \(\zeta\)-regularization theory and give precise statements of both Atiyah’s result and ours.

This is a joint work with Mattia Coloma and Domenico Fiorenza.

References
