

ON NON COEXISTENCE OF 2 & 3-RATIONAL CAUSTICS IN NEARLY CIRCULAR BILLIARD TABLES

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A famous Birkhoff conjecture states that the only integrable convex planar billiards are billiards in an ellipse. We examined two closely related rigidity questions. A rational caustic is a caustic associated to a family of periodic orbits of the same period and the same rotation number. For example, a convex domain with a rational caustic of period two is a domain of a constant width. We investigated a question proposed by Tabachnikov: are there nearly circular domains other than discs with two rational caustics of a prime period p and q ? In this talk, I will discuss our following two new results:

- (rigidity) There are no nearly circular domains with two coexisting rational caustics of period two and three.
- (no super-rigidity) There may be infinitely many deformations of the circular domains with two coexisting rational caustics of period three and five with error given by the 3rd power of the perturbation parameter.

This is based on a joint work with Vadim Kaloshin.