

OPTIMAL GRADIENT REGULARITY FOR SEMILINEAR AND QUASILINEAR EQUATIONS WITH POWER-GROWTH NONLINEARITIES

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In this talk I will survey on recent developments concerning the optimal gradient regularity properties of solutions to some nonlinear elliptic and parabolic equations with first-order terms having power-like growth and unbounded right-hand side in Lebesgue scales. These results are obtained through integral Bernstein methods and/or duality arguments and answer a conjecture raised by P.-L. Lions on stationary problems [9, 10]. In particular, such approaches allow to encompass viscous Hamilton-Jacobi equations, both in the stationary and parabolic case, see [3, 5] and [1, 2] respectively, problems with nonlocal diffusion [6] as well as equations driven by p -Laplacian operators [4]. Finally, I will discuss the implications of these results to the regularity theory of Mean Field Games systems introduced by J.-M. Lasry and P.-L. Lions [8], see [2, 7].

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