CAPACITARY POTENTIALS IN RIEMANNIAN MANIFOLDS AND GEOMETRIC APPLICATIONS

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In this talk I will discuss, in manifolds (M, g) with nonnegative Ricci curvature, monotonicity formulas for suitable integral quantities defined along the level sets of the p-capacitary potential of a bounded $\Omega \subset M$ with smooth boundary. Various analytic/geometric consequences are derived.

The most general purely geometric inequality we obtain is given by the Minkowski Inequality

$$\left(\frac{|\partial\Omega|}{|\mathbb{S}^{n-1}|}\right)^{\frac{n-2}{n-1}} \mathrm{AVR}(g)^{\frac{1}{n-1}} \leq \frac{1}{|\mathbb{S}^{n-1}|} \int_{\partial\Omega} \left|\frac{H}{n-1}\right| d\sigma, \tag{1}$$

for *outward minimizing* domains $\Omega \subset M$, where H is the mean curvature of $\partial\Omega$ and $\mathrm{AVR}(g)$ is the asymptotic volume ratio of (M, g).

Moreover we show that equality holds true if and only if $(M \setminus \Omega, g)$ is isometric to a truncated cone over $\partial\Omega$.

The arguments and the results involve many other important concepts such as isoperimetric/isocapacitary inequalities, outward minimising sets and the Inverse Mean Curvature Flow, that will be briefly discussed.

The talk is mainly based on the papers [1], [2], [3].

References

[1] V. Agostiniani, M. Fogagnolo, L. Mazzieri, *Sharp geometric inequalities for closed hypersurfaces in manifolds with nonnegative Ricci curvature*, Inv. Math. 2020

[2] L. Benatti, M. Fogagnolo, L. Mazzieri, *Minkowski inequalities on complete Riemannian manifolds with nonnegative Ricci curvature*, https://arxiv.org/abs/2101.06063

[3] M. Fogagnolo, L. Mazzieri, *Minimising hulls, p-capacity and isoperimetric inequality on complete Riemannian manifolds*, https://arxiv.org/abs/2012.09490

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