In this talk I will discuss, in manifolds $(M, g)$ with nonnegative Ricci curvature, monotonicity formulas for suitable integral quantities defined along the level sets of the $p$-capacitary potential of a bounded $\Omega \subset M$ with smooth boundary. Various analytic/geometric consequences are derived.

The most general purely geometric inequality we obtain is given by the Minkowski Inequality

\[
\left( \frac{|\partial \Omega|}{|S^{n-1}|} \right)^{\frac{n-2}{n-1}} \text{AVR}(g)^{\frac{1}{n-1}} \leq \frac{1}{|S^{n-1}|} \int_{\partial \Omega} \left| H \right|^n d\sigma,
\]

for outward minimizing domains $\Omega \subset M$, where $H$ is the mean curvature of $\partial \Omega$ and AVR$(g)$ is the asymptotic volume ratio of $(M, g)$.

Moreover we show that equality holds true if and only if $(M \setminus \Omega, g)$ is isometric to a truncated cone over $\partial \Omega$.

The arguments and the results involve many other important concepts such as isoperimetric/isocapacitary inequalities, outward minimizing sets and the Inverse Mean Curvature Flow, that will be briefly discussed.

The talk is mainly based on the papers [1], [2], [3].

References

