INFINITESIMAL DEFORMATIONS AND THE EXTENDED TROPICAL VERTEX GROUP

VERONICA FANTINI

I will discuss the relationship between scattering diagrams and infinitesimal deformations of holomorphic pairs, which Fukaya outlined in [4], and which I studied in my PhD thesis [3].

Scattering diagrams were introduced by Kontsevich and Soibelman in the context of mirror symmetry [7]. They are defined algebraically, in terms of pro-nilpotent Lie groups, but in many applications they have a combinatorial structure which encodes enumerative geometric data (as in [5], [8], [6], [1], [3], for example).

I will cosider a holomorphic pair $(\check{X}, \check{E})$ where $\check{X}$ is the total space of a torus fibration and $\check{E} \to \check{X}$ is an holomorphic vector bundle. My main construction builds on a recent work of Chan, Conan Leung, and Ma [9], where the authors study the relationship between scatteirng diagrams and infinitesimal deformations of $\check{X}$. The new feature introduced in [2] is the extended tropical vertex group $\tilde{\mathbb{V}}$ where the scattering diagrams are defined. It consists of the *large volume limit asymptotics* of the automorphisms that act on deformations of $(\check{X}, \check{E})$. The asymptotics are taken in the limit $\hbar \to 0$, where $\hbar$ is a formal parameter that rescales the complex structure of $\check{X}$. I will sketch the proof of one of the results from [2]:

Theorem 1. *Let $\mathfrak{D}$ be a scattering diagram in $\tilde{\mathbb{V}}$.*

- *One can define an associated 1-form*
$$\Phi_{\mathfrak{D}} = a_1(\hbar)\, t + a_2(\hbar)\, t^2 + a_3(\hbar)\, t^3 + \dots,$$
which is a formal series in t with coefficients in $\Omega^1(\check{X}, \mathrm{T}^{1,0}\check{X} \oplus \mathrm{End}\, \check{E})$.

Suppose $\mathfrak{D}$ consists of two non-parallel walls. Then:

- *The Maurer-Cartan equation which governs deformations of $(\check{X}, \check{E})$ has a unique solution Φ which matches $\Phi_{\mathfrak{D}}$ at first order in t.*
- *The asymptotics of Φ at $\hbar \to 0$ give a saturation of $\mathfrak{D}$.*

References

[1] P. Bousseau, *The quantum tropical vertex*, Geom. Topol. 24 (3) pp. 1297–1379

[2] V. Fantini, *Deformations of holomorphic pairs and 2d-4d wall-crossing*, ATMP, to appear

[3] V. Fantini, *The extended tropical vertex group*, Phd thesis

[4] K. Fukaya, *Multivalued Morse theory, asymptotic analysis and mirror symmetry*, Graphs and patterns in mathematics and theoretical physics, Proc. Sympos. Pure Math., vol. 73, Amer. Math. Soc., Providence, RI, 2005, pp. 205–27

[5] M. Gross, R. Pandharipande, and B. Siebert, The tropical vertex, Duke Math. J. 153, 2010, no.2, pp.297-362.

[6] M. Gross, P. Hacking, and S. Keel, *Mirror symmetry for log Calabi-Yau surfaces I*, Publ. Math. Inst. Hautes Etudes Sci., 122, pp. 65–168, 2015.

[7] M. Kontsevich and Y. Soibelman, *Affine structures and non-Archimedean analytic spaces*, The unity of mathematics, Progr.Math., vol. 244, Birkhäuser Boston, Boston, MA, 2006, pp. 321–385

[8] M. Kontsevich and Y. Soibelman, *Motivic Donaldson-Thomas invariants: summary of results, Mirror symmetry and tropical geometry*, Contemp.Math., vol. 527, Amer.Math. Soc., Providence, RI, 2010, pp. 55–89.

[9] K. Chan, N. Conan-Leung, N. Z. Ma, *Scattering diagrams from asymptotic analysis on Maurer-Cartan equations*, J. Eur. Math. Soc. (JEMS), to appear

IHES
Email address: fantini@ihes.fr