GLASSY BEHAVIOR IN MISMATCHED RANK-ONE MATRIX ESTIMATION

FRANCESCO CAMILLI

This contribution will focus on a topic that has recently attracted attention both from the Statistical Physics and Information Theory communities: the mismatched rank-one matrix estimation. In this setting the Statistician, who has to infer a rank-one matrix blurred by Gaussian additive noise, assumes a *prior* on its elements that does not match the real one. We will show that the problem can be mapped into a particular spin glass model whose free energy is identified by a variational principle on two order parameters: the Parisi overlap distribution and the Mattis magnetization. Once the free energy is given, the main Information Theoretic quantities, such as the Mean Square Error of the matrix estimation, are derived. The specific mismatch of a Bernoulli prior with the product of two Gaussians is analyzed in detail and shown to exhibit a rich behavior, including glassy phases.

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University of Bologna - École Normale Supérieure
Email address: francesco.camilli2@unibo.it, francesco.camilli@ens.psl.eu
URL: https://www.unibo.it/sitoweb/francesco.camilli2