Depth-averaged Finite Volume numerical model for viscous fluids with application to the simulation of lava flows

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We developed a model that may be used for generic viscous fluids. Our new solver simulates free-surface viscous fluids whose temperature changes are due to radiative, convective, and conductive heat exchanges. A temperature-dependent viscoplastic model is used for the final application to lava flows. The model was derived from mass, momentum, and energy conservation laws. The numerical schemes adopted belong to the Finite Volume Methods. This choice allows the creation and propagation of discontinuities in the solutions and enforces the conservation properties of the equations.

Our depth-averaged model describes the dynamics of a viscous fluid in an incompressible and laminar regime. It presents an additional transport equation for a scalar quantity varying horizontally. Viscosity and non-constant vertical profiles for the velocity and the transported quantity are assumed, overtaking the classic shallow-water formulation. In fact, the original assumption that the horizontal velocity field can be considered constant with depth is too restricted when the vertical shear is essential so it must relax, producing a modified momentum equation. As a result, a coefficient, known as the Boussinesq factor, appears in the advective term. The spatial discretization method we employed is a modified version of the central-upwind scheme introduced by Kurganov and Petrova [1] for the classical shallow water equations. For the temporal discretization, we used an implicit-explicit Runge-Kutta technique [2] that permits an implicit treatment of the stiff terms. The whole scheme preserves the positivity of flow thickness and the stationary steady-states [3]. Several numerical experiments validate the proposed method, show the incidence on the numerical solutions of shape coefficients introduced in the model and present the effects of the viscosity-related parameters on the final emplacement of a lava flow.

References

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