THE MORSE INDEX FOR CONSTRAINED OPTIMAL CONTROL PROBLEMS

STEFANO BARANZINI

In this talk I will propose a method to compute the index of the second variation for constrained optimization problems.

Our strategy is to separate the contributions coming from the problem with fixed final and initial state from those related to the boundary conditions. We obtain a formula involving three geometric quantities: the linearisation of a flow on the cotangent bundle (the *extremal flow*), the annihilator of the boundary conditions manifold (which one could think of as a symplectic version of the normal bundle) and the Maslov intersection index.

The latter object is an invariant ubiquitous in the field. Roughly speaking it measures the relative position of the three subspaces.

Our formula is quite versatile and can be employed to study explicit examples (also numerically) and to derive other theoretical results. On one hand we can produce various instances of *iteration formulae* for periodic extremal and *discretization* results to reduce the computation of the index to a finite dimensional problem. On the other hand we can use it to study explicitly some minimization problems, such as the non linear Schrödinger equation on graphs.

This is a joint work with A. Agrachev and I. Beschastnyi.

References

[1] A. Agrachev, S. Baranzini, I. Beschastnyi, *Morse index formulas for minimal graph embeddings*, Submitted to Nonlinearity

SISSA - Scuola Internazionale Superiore di Studi Avanzati, Trieste, Italy
Email address: sbaranzi@sissa.it