INTRINSIC SURFACE FINITE ELEMENT METHOD FOR PDES ON FIXED AND MOVING SURFACES

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From level-set based techniques [1] to the surface finite element method [2] and isogeometric analysis [3], a host of numerical approaches for surface PDEs have been proposed over the last twenty years. Many, like the surface finite element method of [2], rely on an embedding of the surface in a higher dimensional space. These methods have proven successful in applications from fluid flow to biomedical engineering and electromagnetism. We present here an alternative finite element approach based on a geometrically intrinsic formulation [4], that we call Intrinsic Surface Finite Element Method (ISFEM). By careful definition of the geometry and the transport operators, we are able to arrive at an approximation that is fully intrinsic to the surface. We consider first a scalar advection-diffusion-reaction equation defined on a surface. In this case, the numerical analysis of the scheme is also available [5], and we show numerical experiments that support theoretical results. Then, we extend the differential operators for the case of vector-valued partial differential equations. In this case the presented formulation allows the direct discretization of objects naturally defined in the tangent space, without the need of any additional projection. Finally, we extend ISFEM to consider moving surfaces via an intrinsic re-definition of the PDE that takes into account a time-dependent metric tensor. To evaluate our approach, we consider several steady and transient problems involving both diffusion and advection-dominated regimes and compare its performance to established finite element techniques.

This is a joint work with M. Farthing, M. Putti and A. Voigt.

References

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