SEMI-MARKOV PROCESSES, TIME-NONLOCAL EQUATIONS AND RELATED SPECTRAL METHODS

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Since their introduction, Semi-Markov processes played a prominent role in probability theory, in particular when applied to queueing and reliability theory. Different aspects of Semi-Markov processes, however, were considered only in the last years, due to the increasing interest in time-nonlocal integro-differential equations. Analogously to what happens for heat-like equations and strong Markov processes, it has been shown, via semigroup theory, that a class of Semi-Markov processes can be used to describe the solutions of time-nonlocal heat-like equations, in which the classical derivative is substituted by a suitable convolution operator. The main role, in this theory, is played by subordinators and their inverses. The latter can be used to fully describe the eigenfunctions of the aforementioned nonlocal integro-differential operators. Thus, we ask whether such eigenfunctions can substitute the exponential in obtaining spectral decomposition results. Precisely, we consider the case of time-nonlocal Pearson diffusions, that were introduced in [3] for the stable subordinator and then extended in [2] for a wider class of subordinators. The choice is clearly motivated by the tractability of the generators of Pearson diffusions and their link with classical orthogonal polynomials. Similar arguments hold for a suitable class of birth-death processes that arise from the discretization of the Kolmogorov equations of the light-tailed Pearson diffusions, as shown in [1]. This is a joint work with Enrica Pirozzi from Università degli Studi di Napoli "Federico II" and Nikolai Leonenko from University of Cardiff.

References

[1] Giacomo Ascione, Nikolai Leonenko, and Enrica Pirozzi, *Non-local solvable birth–death processes*, Journal of Theoretical Probability, pp. 1–40, 2021.

[2] Giacomo Ascione, Nikolai Leonenko, and Enrica Pirozzi, *Time-non-local Pearson diffusions*, Journal of Statistical Physics, 183(3), pp. 1–42, 2021.

[3] Nikolai Leonenko, Mark Meerschaert, and Alla Sikorskii, *Fractional Pearson diffusions*, Journal of mathematical analysis and applications, 403(2), pp. 532–546, 2013.

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