POLISH-LIKE SPACES AND DESCRIPTIVE SET THEORY AT UNCOUNTABLE CARDINALS

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Descriptive set theory is the study of "definable sets" in Polish (i.e. separable completely metrizable) spaces. Its wide applicability comes from the fact that Polish spaces are ubiquitous in mathematics (and not only there). Classical Descriptive Set Theory has a natural generalization that occurs when countable is replaced by uncountable, called Generalized Descriptive Set Theory. Until recently, Generalized Descriptive Set Theory focused mainly on the study of the generalized Baire space ${}^{\kappa}\kappa$ for a cardinal κ satisfying $\kappa^{<\kappa} = \kappa$, obtaining groundbreaking results (see e.g. the wonderful connection with Shelah's stability theory [7]). However, this framework is really narrow compared to the one of the classical Descriptive Set Theory, focusing on a single space more than on a class of spaces, and heavily relying on cardinal assumptions such as the regularity of κ.

In the last few years, some mathematicians (including myself) worked on a project aimed at filling these gaps by developing a solid theoretical framework consisting of a class of spaces that could take the role of Polish spaces in the uncountable setting [4, 8, 2, 3], and by extending the theory known for cardinals satisfying $\kappa^{<\kappa} = \kappa$ to include more cases, like that of singular cardinals [5, 6, 1].

In this talk, I will present some of the key notions and most relevant ideas on the subject. I will introduce four classes of Polish-like spaces that are suitable for generalized descriptive set theory and explain the relationships between them. Then, I will provide examples of theorems that can be extended from classical descriptive set theory or from ${}^{\kappa}\kappa$ to these classes of spaces. Finally, I will hint how this framework can be extended to singular cardinals.

This is joint work with Luca Motto Ros and Philipp Schlicht.

References

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