SPONTANEOUS BREAKING OF CONTINUOUS SYMMETRY IN THE HEISENBERG MODEL: OLD AND NEW

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In this talk I will first introduce the notion of spontaneous symmetry breaking in statistical mechanics, with particular emphasis on the case of broken continuous symmetry for models of interacting continuous spins, such as the XY model or the Heisenberg model. Then I will review the state of the art, describing some of the most important and influential results on the low temperature behavior of such systems, most notably the one of Frohlich-Simon-Spencer, who proved, in 1976, the existence of orientational long range order for the 3D classical Heisenberg model via the first application of reflection positivity methods to statistical mechanics, and the one of Dyson-Lieb-Simon, who extended this result, in 1978, to the case of the quantum anti-ferromagnetic Heisenberg model. Next I will discuss some important open problems and review recent advances on the understanding of the low temperature behavior of classical and quantum Heisenberg models in three dimensions.