Unione Matematica Italiana

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Università degli Studi di Padova

DIPARTIMENTO MATEMATICA
DIPARTIMENTO DI MATEMATICA "TULLIO LEVI-CIVITA"

Book of Abstracts

in alphabetical order

UMI 100 UniPD

Padova, 23rd-27th May 2022

Welcome from
Organizing Committee and Scientific Committees Chairs

Welcome to Padova and to 100 UMI - 800 UniPD
"100 Years Unione Matematica Italiana - 800 Years Università di Padova"!

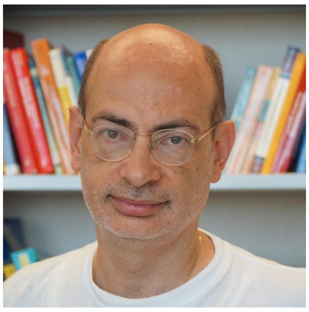
In 2022, the Italian Mathematical Union (UMI) and the University of Padova celebrate the 100th anniversary and the 800th anniversary of their founding, respectively. On this unique occasion, UMI and the Department of Mathematics of University of Padova have organized an international conference hosting a world renowned list of plenary speakers and panelists. The conference hosts also the "First UMI Meeting of PhD students".

We have done our best to offer a rich program with several and diversified points of attraction. So we sincerely hope that you will find the plenary talks a rewarding experience and the three roundtables a useful opportunity to look into the past, present and future of our community. We have also included in the program two occasions of "mathematical" entertainment involving comics and drama. Last but not least, we welcome with pride the presence of over 110 young speakers coming from all over Italy. They represent the future of mathematics in our country.

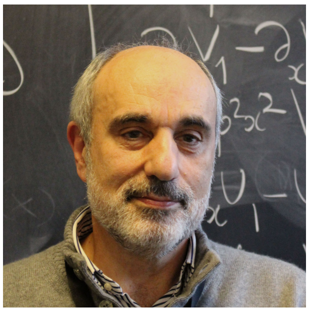
We would like to take this opportunity to thank the University of Padova and Rettrice Prof. Daniela Mappelli. Moreover, we would like to thank Prorettore Prof. Marco Ferrante for all his support and helps in organizing this event. We thank as well the Director of the Department of Mathematics Prof. Bruno Chiarellotto. We would like to express our gratitude to Prof. Francesco Rossi and to all members of the Organizing and Scientific Committees who contributed essentially to the success of the final program of this conference. Special thanks are due to Cinzia Clemente and to the administrative and technical staff of the Department of Mathematics, as well as to Gioia Grigolin and to the staff of the area communication and events, for their dedication and professional work in helping to organize every logistic aspect of this conference. We thank also Elisabetta Velabri, Milena Tansini Pagani and Marcello Villani of the staff of UMI for all their valuable contribution.

We look forward to your participation in a stimulating and productive environment of the 100 UMI - 800 UniPD conference.

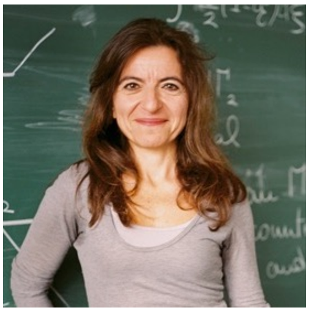
Fabio Ancona



Piermarco Cannarsa



Barbara Nelli



Book of Abstracts
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Monday 23rd - Aula Magna di Palazzo Bo

Tuesday 24th - Aula Magna di Palazzo Bo

Wednesday 25th - Auditorium Pollini

Thursday 26th - Torre Archimede

09:00* **Alessandro Giuliani** - *Spontaneous breaking of continuous symmetry in the Heisenberg model: old and new* (Chair: B. Nelli) . . . 15
10:00 **Parallel sessions - BDEFGJKLMN**
11:00 Coffee break - Ground Floor
11:30 **Parallel sessions - BDEFGJKLMN**
12:30 Lunch
14:00* **Andrea Mondino** - *Smooth and non-smooth aspects of Ricci curvature lower bounds* (Chair: D. Vittone) . . . 16
15:00 Coffee break - Ground Floor
15:30 **Parallel sessions - ACDEGJKLMN**
16:30* **Round Table "11^2 years after Volterra: applying mathematics to biological and social sciences"**: I. Dorigatti, M. Fornasier, B. Piccoli (Chair: R. Natalini)

Friday 27th - Torre Archimede

08:30 **Parallel sessions - EGJM**
09:00 **Parallel sessions - ABCEGHJLM**
09:30* **Giulia Saccà** - *Holomorphic symplectic manifolds and completely integrable systems* (Chair: B. Nelli) . . . 17
10:30 Coffee break - Ground Floor
11:00* **The after math. Quali lavori per i PhD in Matematica? Incontro con alcune aziende** (in Italian, Chairs: G. Callegaro, A. Larese)
12:00 **Parallel sessions - ABCDFGHIKM**
13:00 Lunch
14:30 **Parallel sessions - ABCDFGIKLM**
15:30* **Daniele Di Pietro** - *From physical models to advanced numerical methods through de Rham cohomology* (Chair: S. De Marchi) . . . 18
16:30 Farewell Spritz - Ground Floor

* In Torre Archimede, all Plenaries and Round Tables are in Room 1A150 - 1st Floor, with live streaming in Room 1C150 - 1st Floor.

ON THE COMPLEX COBORDISM CLASSES OF HYPER-KÄHLER MANIFOLDS

CLAIRE VOISIN

Hyper-Kähler manifolds are symplectic holomorphic compact Kähler manifolds, a particular class of complex manifolds with trivial canonical bundle. They exist only in even complex dimension, and there are two main series of known deformation classes of hyper-Kähler manifolds, with one model in each even dimension, that I will describe. I will discuss in this introductory talk a result obtained with Georg Oberdieck and Jieao Song on the complex cobordism classes of hyper-Kähler manifolds, and present a number of open questions concerning their Chern numbers.

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A MATHEMATICAL JOURNEY THROUGH SCALES

MARTIN HAIRER

The tiny world of particles and atoms and the gigantic world of the entire universe are separated by about forty orders of magnitude. As we move from one to the other, the laws of nature can behave in drastically different ways, sometimes obeying quantum physics, general relativity, or Newtons classical mechanics, not to mention other intermediate theories. Understanding the transformations that take place from one scale to another is one of the great classical questions in mathematics and theoretical physics, one that still hasn't been fully resolved. In this lecture, we will explore how these questions still inform and motivate interesting problems in probability theory and why so-called toy models, despite their superficially playful character, can sometimes lead to certain quantitative predictions.

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THE UNIONE MATEMATICA ITALIANA IN THE INTERWAR PERIOD. SCIENTIFIC, INSTITUTIONAL AND POLITICAL ASPECTS

LIVIA GIACARDI

The Italian Mathematical Union (UMI) celebrates its centenary this year. It was indeed set up in 1922 in accordance with a motion approved in Brussels in July 1919 by the International Research Council, which promoted the creation of national scientific committees. The national and international background of this event is very problematic in various respects. 1922 is the year of the March on Rome that brought the fascist party to power, and later on to a gradual transformation of the fascist government into a dictatorship. Furthermore, serious international tensions made the situation even more complex. In particular, in the aftermath of WWI the ex-Central Powers were excluded from the new scientific institutions the International Research Council (1919) and the International Mathematical Union (1920). The recent reorganization of the UMI Archives has made a lot of significant documents available to historians, who can now shed light on the backstage of the first twenty years of UMI's life.

We aim to outline UMI's history in the interwar period by considering three different aspects: autarky versus internationalism; pure mathematics versus applied mathematics; fascist policy versus circulation of people and mathematical ideas. We will focus on the (i) role of Volterra and Pincherle in the foundation of the UMI and certain initial difficulties; (ii) UMI's international relationships and especially Pincherle's involvement in the International Congresses of Mathematicians of Toronto (1924) and of Bologna (1928); (iii) UMI's relations with fascism and their consequences on its Bulletin. Our research is based on unpublished letters and documents contained in the UMI Archives.

This is a joint work with ROSSANA TAZZIOLI (rossana.tazzioli@univ-lille.fr)

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RIGIDITY AND UNIFORMITY IN ALGEBRAIC DYNAMICS

LAURA DE MARCO

The periodic orbits and their structure are fundamental features of a dynamical system. In an algebraic setting, where the system is defined by polynomials, we can use tools from algebraic or arithmetic geometry to study these orbits. Important special cases include endomorphims of abelian varieties, for example as appearing in the proofs of uniform versions of the Mordell or Manin-Mumford Conjectures in the recent breakthroughs of Dimitrov-Gao-Habegger, Kühne, Yuan and others, where the torsion points of the group coincide with the preperiodic points of an endomorphism. In this talk, I will describe some parallel questions and recent progress on more general families of complex and arithmetic dynamical systems.

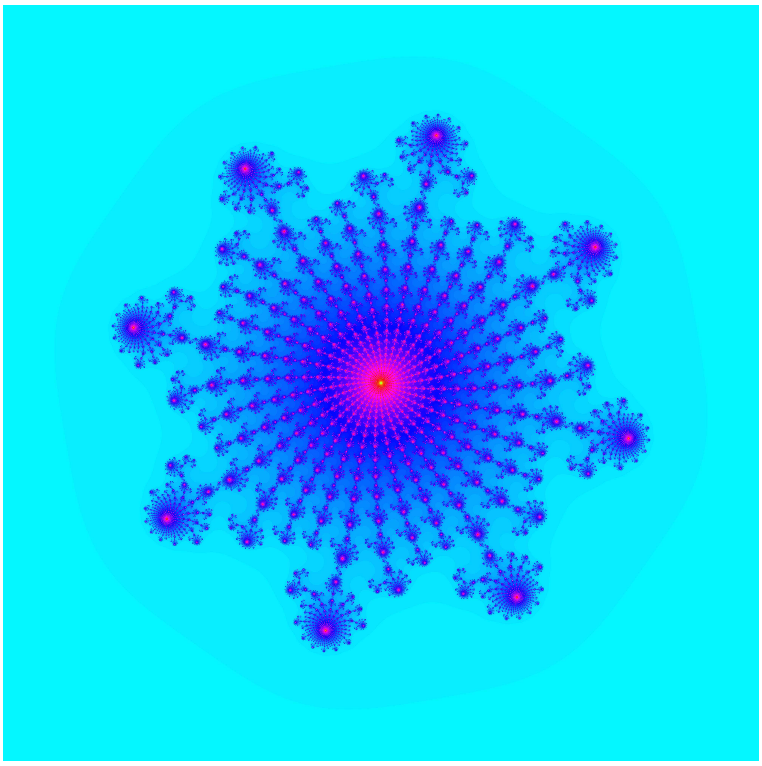

FIGURE 1. The Julia set of a polynomial with symmetries

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PADOVA AND THE ABSOLUTE DIFFERENTIAL CALCULUS

LUCA DELL'AGLIO

The aim of this talk is to highlight the close connections between the University of Padova and the history of the absolute differential calculus, particularly in the period preceding its consideration as the mathematical theory of general relativity. This concerns both the origins and early developments of tensor analysis, above all in relation to the central and differentiated roles played in this context by the research of Gregorio Ricci-Curbastro and Tullio Levi-Civita. Furthermore, the difficulties that the absolute differential calculus found at the time in terms of reception, especially at the national level, are of particular relevance here, being partly accepted from some points of view but not from others. A factor that further accentuated the role played by the scientific context of Padova in the initial phase of the theory's development.

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LAGRANGIAN SYSTEMS CONTROLLED BY ACTIVE CONSTRAINTS

ALBERTO BRESSAN

The talk will survey various results on the control of mechanical systems, by means of time-dependent, frictionless constraints. The basic mathematical description involves a Riemann manifold, together with a foliation describing the constraints. The equations of motion usually have an impulsive character, containing the time derivative of the control function. Their analytical form is closely linked to the geometric structure of the foliation. This same framework can also be used to study swim-like motion of one or more deformable bodies in a perfect fluid.

Major contributions to this theory were provided by researchers from the University of Padova.

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CONDENSED MATHEMATICS

PETER SCHOLZE

One of the most basic notions in mathematics is the notion of a topological space, which formalizes the idea of a space with a notion "nearness" of points. First introduced by Hausdorff in 1914, it has become central in all areas of mathematics. However, in some respects the notion of topological space is not optimal; for example, it can not formalize the idea of "points that are infinitely near but distinct" in a useful way. In 2018, Dustin Clausen came to Bonn, and proposed a certain substitute for topological spaces that we termed condensed sets, and that overcomes these foundational issues. I will try to give an overview of these ideas.

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BOUNDARY REGULARITY OF MINIMAL SURFACES

CAMILLO DE LELLIS

The critical points of the area functional, usually called minimal surfaces, have a long history in mathematics. Perhaps the most famous examples are the solutions of the so-called Plateau's problem, i.e. surfaces which minimize the area among the ones spanning a given contour. It is known since long that area minimiziers can form singularities and several concepts of generalized solutions, which serve different purposes, have been introduced in the literature since the first decades of the last century. A wide field of study is the regularity of the latter objects. While there is a quite good understanding of the size of singularities away from the boundary in very many situations, the same cannot be said for the case of boundary singularities, for which we have very satisfactory theorems only in relatively few, albeit important, cases. I will review some results of the last decade which touched for the first time a category of problems in the area, and I will explain a recent joint work with Stefano Nardulli and Simone Steinbruchel which gives a first positive answer to a question of Allard and White.

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OUTCOME INDISTINGUISHABILITY, SCAFFOLDING SETS, AND PAN-CALIBRATION

CYNTHIA DWORK

Prediction algorithms score individuals, or individual instances, assigning to each one a number in the range from 0 to 1. That score is often interpreted as a probability: What are the chances that this loan will be repaid? How likely is this tumor to metastasize? A key question lingers: What is the "probability" of a non-repeatable event? This is the defining problem of AI. Without a satisfactory answer, how can we even specify what we want from an ideal algorithm?

This talk will introduce *outcome indistinguishability* [2], a desideratum with roots in computational complexity theory, and will situate the concept within the landscape of algorithmic fairness.

Outcome indistinguishability generalizes *multi-calibration*, a fairness notion for prediction algorithms that requires simultaneous calibration on a (possibly large) pre-specified collection of subsets of the population [3]. Here, too, a question lingers: what can be done to ensure that all subordinated groups – including those whose members cannot advocate for themselves – are included in the collection?

We will show how to circumvent this problem through the use of a *Scaffolding Set* collection [1], and give some simple conditions under which such a collection can be efficiently constructed. When these conditions are not met, no harm is done; when they are satisfied, calibration is achieved simultaneously on all large subpopulations, a concept we call *pan-calibration*.

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UBIQUITÀ DEL TRASPORTO OTTIMALE

ALESSIO FIGALLI

Alla fine del XVIII secolo, Gaspard Monge introdusse il trasporto ottimale come strumento per capire il modo più efficiente di trasportare una distribuzione di materiale da un luogo all'altro per costruire fortificazioni. Più tardi, negli anni '40, Kantorovich svilupp questa teoria e per il suo lavoro ricevette il premio Nobel per l'economia. Negli ultimi 30 anni il trasporto ottimale ha trovato varie applicazioni in molti problemi, sia matematici che di natura più applicata. In questa presentazione darò una panoramica di questa teoria e di alcune delle sue applicazioni.

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SPONTANEOUS BREAKING OF CONTINUOUS SYMMETRY IN THE HEISENBERG MODEL: OLD AND NEW

ALESSANDRO GIULIANI

In this talk I will first introduce the notion of spontaneous symmetry breaking in statistical mechanics, with particular emphasis on the case of broken continuous symmetry for models of interacting continuous spins, such as the XY model or the Heisenberg model. Then I will review the state of the art, describing some of the most important and influential results on the low temperature behavior of such systems, most notably the one of Frohlich-Simon-Spencer, who proved, in 1976, the existence of orientational long range order for the 3D classical Heisenberg model via the first application of reflection positivity methods to statistical mechanics, and the one of Dyson-Lieb-Simon, who extended this result, in 1978, to the case of the quantum anti-ferromagnetic Heisenberg model. Next I will discuss some important open problems and review recent advances on the understanding of the low temperature behavior of classical and quantum Heisenberg models in three dimensions.

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SMOOTH AND NON-SMOOTH ASPECTS OF RICCI CURVATURE LOWER BOUNDS

ANDREA MONDINO

After recalling the basic notions coming from differential geometry, the talk will be focused on spaces satisfying Ricci curvature lower bounds. The idea of compactifying the space of Riemannian manifolds satisfying Ricci curvature lower bounds goes back to Gromov in the 80s and was pushed by Cheeger and Colding in the 90s who investigated the fine structure of possibly non-smooth limit spaces. A completely new approach via optimal transportation was proposed by Sturm and Lott-Villani around fifteen years ago. Via such an approach one can give a precise definition of what means for a non-smooth space to have Ricci curvature bounded below. Such an approach has been refined in the last years giving new insights to the theory and yielding applications which seems to be new even for smooth Riemannian manifolds.

The talk is meant to be an introduction to the topic, accessible to non-specialists and as self-contained as possible.

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HOLOMORPHIC SYMPLECTIC MANIFOLDS AND COMPLETELY INTEGRABLE SYSTEMS

GIULIA SACCÀ

Irreducible holomorphic symplectic manifolds are one of the building blocks of compact Kähler manifolds with trivial first Chern class. They made their first appearance in algebraic geometry in the 80's [1, 3], thanks to results in differential geometry, and since then have attracted significant attention. Their rich geometry has ties to other areas of mathematics, such as representation theory and mathematical physics.

Some irreducible holomorphic symplectic manifolds have a structure of completely integrable system which, in this context, means that they admit a fibration whose general fiber is a complex torus which is Lagrangian [2]. These fibrations are called Lagrangian and it turns out they are extremely useful for constructing and studying examples of irreducible holomorphic symplectic manifolds. In this talk I will give an introduction to irreducible holomorphic symplectic manifolds, with a focus on Lagrangian fibrations.

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FROM PHYSICAL MODELS TO ADVANCED NUMERICAL METHODS THROUGH DE RHAM COHOMOLOGY

DANIELE A. DI PIETRO

The well-posedness of relevant physical models expressed in terms of partial differential equations (PDE) hinges on subtle analytical, homological, and algebraic properties underlying Hilbert complexes [1]. The best-known example is the de Rham complex which, in practically relevant situations, can be expressed through vector proxies as the sequence of Hilbert spaces H^1, $\boldsymbol{H}(\mathbf{curl})$, $\boldsymbol{H}(\mathrm{div})$, and L^2 connected by the vector calculus operators gradiend, curl, and divergence. In the first part of this presentation, we illustrate the role of the de Rham complex in the well-posedness of several PDE problems arising in various fields of physics.

The design of efficient numerical methods for such problems is challenging for several reasons: on one hand, stability requires to mimic, at the discrete level, the homological and analytical properties of the de Rham complex (leading to the notion of "compatible method"); on the other hand, the complicated geometrical features of the domain and behaviours of the solution require a great flexibility in terms of supported meshes and approximation orders. In the second part of this presentation we provide an introduction to the recently introduced Discrete de Rham (DDR) paradigm [4, 2, 3] for the design and analysis of compatible discretization methods supporting general polyhedral meshes and arbitrary orders.

The general principle of DDR methods is to replace both spaces and operators by discrete counterparts designed so as to be compatible with the cohomology properties of the continuous complex. Specifically:

- The discrete spaces are spanned by vectors of polynomials with components attached to mesh entities in order to mimic, through their single-valuedness, global (complete or partial) continuity properties of the continuous spaces. The local polynomial spaces can be either full or incomplete.
- The discrete operators are obtained in two steps: first, reconstructions in full polynomial spaces are built mimicking an approximate version of the Stokes formula; second, whenever needed, the L^2-orthogonal projection on the appropriate incomplete polynomial space is taken.

A full set of results for DDR methods have been recently proved [2], including cohomology-related properties, Poincar inequalities, as well as primal and adjoint consistency of the discrete vector calculus operators. An overview of such results, along with examples of applications, is provided.

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Speakers in Parallel Sessions - Alphabetical order

DECAY ESTIMATES IN EVOLUTION EQUATIONS WITH CLASSICAL AND FRACTIONAL TIME-DERIVATIVES

ELISA AFFILI

Using energy methods, we prove some power-law and exponential decay estimates for classical and nonlocal evolutionary equations. The results obtained are framed into a general setting, which comprise, among the others, equations involving both standard and Caputo time-derivative, and diffusion operators as the classic and fractional Laplacian, complex valued magnetic operators, fractional porous media equations and nonlocal Kirchhoff operators. Both local and fractional space diffusion are taken into account, possibly in a nonlinear setting. The different quantitative behaviours, which distinguish polynomial decays from exponential ones, depend heavily on the structure of the time-derivative involved in the equation. This work was done in collaboration with Enrico Valdinoci [1].

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POLISH-LIKE SPACES AND DESCRIPTIVE SET THEORY AT UNCOUNTABLE CARDINALS

CLAUDIO AGOSTINI

Descriptive set theory is the study of "definable sets" in Polish (i.e. separable completely metrizable) spaces. Its wide applicability comes from the fact that Polish spaces are ubiquitous in mathematics (and not only there). Classical Descriptive Set Theory has a natural generalization that occurs when countable is replaced by uncountable, called Generalized Descriptive Set Theory. Until recently, Generalized Descriptive Set Theory focused mainly on the study of the generalized Baire space ${}^{\kappa}\kappa$ for a cardinal κ satisfying $\kappa^{<\kappa} = \kappa$, obtaining groundbreaking results (see e.g. the wonderful connection with Shelah's stability theory [7]). However, this framework is really narrow compared to the one of the classical Descriptive Set Theory, focusing on a single space more than on a class of spaces, and heavily relying on cardinal assumptions such as the regularity of κ.

In the last few years, some mathematicians (including myself) worked on a project aimed at filling these gaps by developing a solid theoretical framework consisting of a class of spaces that could take the role of Polish spaces in the uncountable setting [4, 8, 2, 3], and by extending the theory known for cardinals satisfying $\kappa^{<\kappa} = \kappa$ to include more cases, like that of singular cardinals [5, 6, 1].

In this talk, I will present some of the key notions and most relevant ideas on the subject. I will introduce four classes of Polish-like spaces that are suitable for generalized descriptive set theory and explain the relationships between them. Then, I will provide examples of theorems that can be extended from classical descriptive set theory or from ${}^{\kappa}\kappa$ to these classes of spaces. Finally, I will hint how this framework can be extended to singular cardinals.

This is joint work with Luca Motto Ros and Philipp Schlicht.

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OPINION DYNAMICS: CONFORMIST AND NONCONFORMIST INTERACTING AGENTS

MICHELE ALEANDRI

We study two models of binary decisions in a connected network of interacting agents. Individual decisions are determined by social influence, coming from direct interactions with neighbours, and a group level pressure that accounts for social environment. We study the convergence of the mean field variables associated to the processes as the number of agents goes to infinity and we show that propagation of chaos occurs.
In the first model, [2], we have a family of conformist or nonconformist agents that interact with each other. When the number of agents is large but fixed, we study the amount of time spent by the mean field variables associated to the process around the stable points of the macroscopic dynamics. In a nonconformist environment, there is a persistent disordered phase where no majority is formed: We show how in this case the introduction of a delay mechanism in the agent's detection of the global average choice may drastically change this scenario, giving rise to a coordinated self sustained periodic behavior.
In the second model, [1], the population is divided into two social groups, each one characterized by its attitude with respect to the other. Agents of the same group interact with each other, while the other group exerts on them a social influence, that may also be null or even negative. We focus in particular on models with Lotka-Volterra type interactions, i.e., models with conformist vs. nonconformist groups. For these models, although the microscopic system is driven a.s. to consensus within each group, a periodic behaviour arises in the macroscopic scale. In order to describe fluctuations between the limiting periodic orbits, we identify a slow variable in the microscopic system and, through an averaging principle, we find a diffusion which describes the macroscopic dynamics of such variable on a larger time scale.
This is a joint work with Ida Germana Minelli.

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CONVERGENCE OF THE PIECEWISE ORTHOGONAL COLLOCATION FOR PERIODIC SOLUTIONS OF DELAY EQUATIONS

ALESSIA ANDÒ

This is a joint work with Dimitri Breda.

I will present an analysis of the convergence of (a variant of) the piecewise orthogonal collocation for periodic solutions of retarded functional differential equations (RFDEs) or renewal equations (REs) defined by a generic right-hand side [1]. Such analysis is highly based on [3] where a general framework for solving a certain class of boundary value problems (BVPs) is presented and accompanied by a rigorous proof of convergence of the corresponding iterative method. The novel contributions consist in the proofs of the validity of the assumptions required to apply the abstract approach of [3] in the case of periodic BVPs. Indeed, although the general BVP in [3] considers the presence of unknown parameters, it does not explicitly deal with the periodic case. In the presentation I will highlight the role of the period as the (main) unknown parameter of the problem, which leads to some effort in validating the required assumptions, being it directly linked to the course of time. It also affects the regularity that must be required from the functionals involved, as well as the choice of the relevant spaces where the solution, its derivative or the states must lie. I will conclude the presentation with some comments on the differences between the case of RFDEs and that of REs, the relevant convergence analysis of which is part of the work [2], currently under review. Despite the similarities in the structure with respect to that of the proof for RFDEs, some more work is required to complete the analysis for REs, which included recurring to some resolvent theory. The further extension to coupled systems can also be carried out by exploiting the ideas already used for RFDEs and REs separately.

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SEMI-MARKOV PROCESSES, TIME-NONLOCAL EQUATIONS AND RELATED SPECTRAL METHODS

GIACOMO ASCIONE

Since their introduction, Semi-Markov processes played a prominent role in probability theory, in particular when applied to queueing and reliability theory. Different aspects of Semi-Markov processes, however, were considered only in the last years, due to the increasing interest in time-nonlocal integro-differential equations. Analogously to what happens for heat-like equations and strong Markov processes, it has been shown, via semigroup theory, that a class of Semi-Markov processes can be used to describe the solutions of time-nonlocal heat-like equations, in which the classical derivative is substituted by a suitable convolution operator. The main role, in this theory, is played by subordinators and their inverses. The latter can be used to fully describe the eigenfunctions of the aforementioned nonlocal integro-differential operators. Thus, we ask whether such eigenfunctions can substitute the exponential in obtaining spectral decomposition results. Precisely, we consider the case of time-nonlocal Pearson diffusions, that were introduced in [3] for the stable subordinator and then extended in [2] for a wider class of subordinators. The choice is clearly motivated by the tractability of the generators of Pearson diffusions and their link with classical orthogonal polynomials. Similar arguments hold for a suitable class of birth-death processes that arise from the discretization of the Kolmogorov equations of the light-tailed Pearson diffusions, as shown in [1]. This is a joint work with Enrica Pirozzi from Università degli Studi di Napoli "Federico II" and Nikolai Leonenko from University of Cardiff.

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INTRINSIC SURFACE FINITE ELEMENT METHOD FOR PDES ON FIXED AND MOVING SURFACES

ELENA BACHINI

From level-set based techniques [1] to the surface finite element method [2] and isogeometric analysis [3], a host of numerical approaches for surface PDEs have been proposed over the last twenty years. Many, like the surface finite element method of [2], rely on an embedding of the surface in a higher dimensional space. These methods have proven successful in applications from fluid flow to biomedical engineering and electromagnetism. We present here an alternative finite element approach based on a geometrically intrinsic formulation [4], that we call Intrinsic Surface Finite Element Method (ISFEM). By careful definition of the geometry and the transport operators, we are able to arrive at an approximation that is fully intrinsic to the surface. We consider first a scalar advection-diffusion-reaction equation defined on a surface. In this case, the numerical analysis of the scheme is also available [5], and we show numerical experiments that support theoretical results. Then, we extend the differential operators for the case of vector-valued partial differential equations. In this case the presented formulation allows the direct discretization of objects naturally defined in the tangent space, without the need of any additional projection. Finally, we extend ISFEM to consider moving surfaces via an intrinsic re-definition of the PDE that takes into account a time-dependent metric tensor. To evaluate our approach, we consider several steady and transient problems involving both diffusion and advection-dominated regimes and compare its performance to established finite element techniques.

This is a joint work with M. Farthing, M. Putti and A. Voigt.

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SECOND-ORDER COVARIATION: AN ANALYSIS OF STUDENTS' FORMS OF REASONING AND TEACHER'S INTERVENTIONS WHEN MODELLING REAL PHENOMENA

SARA BAGOSSI

This research project aims to investigate covariational reasoning understood not only as the ability to visualize two or more magnitudes while co-varying simultaneously [1], but in a broader epistemological sense, as the ability to grasp relationships of invariance between mathematical objects. The need to better characterize more complex forms of reasoning performed by students in mathematical modelling activities led us to introduce second-order covariation, a form of covariation that consists in describing relations in which not only variables are involved but also parameters [2]. These enable to represent families of relationships between variables that is classes of real phenomena characterized, from a mathematical standpoint, through parameters, which determine the specificities of the mathematical model. The discussion of this theme arises not only from research needs in the field of Mathematics Education, i.e., the existence of a theoretical framework only partially useful to describe the covariational reasoning of students, but above all by its relevance in terms of teaching practices. There is a wide literature showing that in mathematical modelling situations the ability to reason covariationally is essential because it allows to envision the invariant relationships that exist between quantities involved in dynamic situations. The *Indicazioni Nazionali* for teaching mathematics in high schools [3] underline the relevance of introducing mathematical modelling as representation of classes of real phenomena. However, despite the acknowledged relevance of covariation for the learning of numerous mathematical concepts, in the Italian mathematics curriculum as well as in most textbooks, explicit references to this approach are generally absent. Teachers themselves have little specific knowledge of covariation and therefore struggle to introduce it into their teaching practices. Data analyzed in this study are from three teaching experiments conducted in the first grades of a scientific-oriented secondary school; their aim was the mathematical description of some real situations: the motion of a ball along an inclined plane [4] and the relationship between temperature and humidity described in the so-called psychrometric diagram. Using appropriate technological tools, students were guided in obtaining a mathematical formula describing such phenomena and in recognizing the different role played by variables and parameters in the writing and reading of different registers of mathematical representation. Students' reasoning processes and the evolution of the different semiotic aspects (spoken, gestural, representational) involved in the teaching-learning processes were analyzed; as well the support of technological tools and the role of the teacher in enhancing covariational reasoning through appropriate adaptive teaching strategies were considered. Eventually, this study led us to the elaboration of a broader theoretical framework about covariation.

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THE MORSE INDEX FOR CONSTRAINED OPTIMAL CONTROL PROBLEMS

STEFANO BARANZINI

In this talk I will propose a method to compute the index of the second variation for constrained optimization problems.

Our strategy is to separate the contributions coming from the problem with fixed final and initial state from those related to the boundary conditions. We obtain a formula involving three geometric quantities: the linearisation of a flow on the cotangent bundle (the *extremal flow*), the annihilator of the boundary conditions manifold (which one could think of as a symplectic version of the normal bundle) and the Maslov intersection index.

The latter object is an invariant ubiquitous in the field. Roughly speaking it measures the relative position of the three subspaces.

Our formula is quite versatile and can be employed to study explicit examples (also numerically) and to derive other theoretical results. On one hand we can produce various instances of *iteration formulae* for periodic extremal and *discretization* results to reduce the computation of the index to a finite dimensional problem. On the other hand we can use it to study explicitly some minimization problems, such as the non linear Schrödinger equation on graphs.

This is a joint work with A. Agrachev and I. Beschastnyi.

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UNITARIZATION AND INVERSION FORMULAE FOR THE RADON TRANSFORM BETWEEN DUAL PAIRS

FRANCESCA BARTOLUCCI

The Radon transform has its origin in the problem of recovering a function defined on $\mathbb{R}^d$ from its integrals over hyperplanes. In 1917 Radon proved the reconstruction formula for two and three-dimensional functions. In $\mathbb{R}^3$ it reads

$$f = -\frac{1}{2}\Delta\mathcal{R}^{\#}\mathcal{R}f, \tag{1}$$

where Δ denotes the Laplacian, $\mathcal{R}$ denotes the Radon transform, and $\mathcal{R}^{\#}$ denotes the dual Radon transform, or back-projection. The Radon transform maps a function on $\mathbb{R}^d$ into the set of integrals over all hyperplanes, while the dual Radon transform maps a function defined on the set of hyperplanes of $\mathbb{R}^d$ into its integrals over the sheaves of hyperplanes through a point. This classical inverse problem is a particular case of the more general issue of recovering an unknown function on a manifold by means of its integrals over a family of submanifolds, already investigated by Gelfand in the 1950s. A natural framework for such general inverse problems was considered by Helgason [2]. He introduced the generalized Radon transform associated to dual pairs $(G/K, G/H)$ of homogeneous spaces of the same locally compact group G, where K and H are closed subgroups of G. The space G/K is meant to describe the ambient in which the functions to be analysed live. The second space G/H is meant to parametrise the set of submanifolds of G/K over which one wants to integrate functions. Every element $\xi \in G/H$ defines a subset $\hat{\xi} \subseteq G/K$. The generalized Radon transform $\mathcal{R}$ takes functions on G/K into functions on G/H and is abstractly defined by

$$\mathcal{R}f(\xi) = \int_{\hat{\xi}} f(x)\mathrm{d}m_\xi(x),$$

where, for all $\xi \in G/H$, m_ξ is a suitable measure on the manifold $\hat{\xi}$ and where f is such that the right hand side is meaningful, possibly in some weak sense. In this context, the most relevant issues are to generalize formula (1) and to prove that the Radon transform, up to a composition with a suitable pseudo-differential operator, can be extended to a unitary operator from $L^2(G/K)$ to $L^2(G/H)$. We prove that if the quasi regular representations π of G on $L^2(G/K)$ and $\hat{\pi}$ of G on $L^2(G/H)$ are irreducible, then the generalized Radon transform $\mathcal{R}$, up to composition with a suitable pseudo-differential operator, can be extended to a unitary operator $\mathcal{Q} : L^2(G/K) \to L^2(G/H)$ intertwining the two representations. If, in addition, the representations are square integrable, we derive an inversion formula for the generalized Radon transform based on the voice transform associated to these representations.

This is a joint work with Giovanni S. Alberti, Filippo De Mari and Ernesto De Vito. See [1].

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PATTERN FORMATION IN SOFT MATTER: EMERGENCE OF FARADAY INSTABILITY IN SOFT SOLIDS

GIULIA BEVILACQUA

Recent experiments have observed the emergence of standing waves at the free surface of elastic bodies attached to a rigid oscillating substrate and subjected to critical values of forcing frequency and amplitude [6, 5]. This phenomenon, known as Faraday instability, is now well understood for viscous fluids [2], where primarily subharmoninc response dominates, but surprisingly eluded any theoretical explanation for soft solids. Recently, the elastic behaviour of the medium has been found to have a dramatic regularizing effect on some well-known dynamic phenomena in fluid mechanics, such as Rayleigh-Plateau [3] or Rayleigh-Taylor instabilities [4]. In this talk, we characterize Faraday waves in soft incompressible slabs using the Floquet theory to study the onset of harmonic and subharmonic resonance eigenmodes. We consider a ground state corresponding to a finite homogeneous deformation of the elastic slab. We transform the incremental boundary value problem into an algebraic eigenvalue problem characterized by the three dimensionless parameters, that characterize the interplay of gravity, capillary and elastic waves. Remarkably, we found that Faraday instability in soft solids is characterized by a harmonic resonance in the physical range of the material parameters. This seminal result [1] is in contrast to the subharmonic resonance that is known to characterize viscous fluids, and opens the path for using Faraday waves for a precise and robust experimental method that is able to distinguish solid-like from fluid-like responses of soft matter at different scales.

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Depth-averaged Finite Volume numerical model for viscous fluids with application to the simulation of lava flows

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We developed a model that may be used for generic viscous fluids. Our new solver simulates free-surface viscous fluids whose temperature changes are due to radiative, convective, and conductive heat exchanges. A temperature-dependent viscoplastic model is used for the final application to lava flows. The model was derived from mass, momentum, and energy conservation laws. The numerical schemes adopted belong to the Finite Volume Methods. This choice allows the creation and propagation of discontinuities in the solutions and enforces the conservation properties of the equations.

Our depth-averaged model describes the dynamics of a viscous fluid in an incompressible and laminar regime. It presents an additional transport equation for a scalar quantity varying horizontally. Viscosity and non-constant vertical profiles for the velocity and the transported quantity are assumed, overtaking the classic shallow-water formulation. In fact, the original assumption that the horizontal velocity field can be considered constant with depth is too restricted when the vertical shear is essential so it must relax, producing a modified momentum equation. As a result, a coefficient, known as the Boussinesq factor, appears in the advective term. The spatial discretization method we employed is a modified version of the central-upwind scheme introduced by Kurganov and Petrova [1] for the classical shallow water equations. For the temporal discretization, we used an implicit-explicit Runge-Kutta technique [2] that permits an implicit treatment of the stiff terms. The whole scheme preserves the positivity of flow thickness and the stationary steady-states [3]. Several numerical experiments validate the proposed method, show the incidence on the numerical solutions of shape coefficients introduced in the model and present the effects of the viscosity-related parameters on the final emplacement of a lava flow.

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MATHEMATICAL MEMES: FROM INTERNET PHENOMENON TO DIGITAL EDUCATIONAL RESOURCE

GIULIA GIOVANNA MARIA BINI

This thesis aims at providing insights into the process of transforming the Internet phenomenon of mathematical memes into a digital educational resource, with the purpose of contributing to bridging the discontinuity between out-of-school and in-school learning environments. Mathematical Internet memes are a nearly unexplored research topic in the field of Mathematics Education. This condition requested to sample theoretical foundations and develop appropriate methodologies, guided by the main research question asking how can we conceptualize mathematical Internet memes and what could their educational potentialities be. The investigation process progressed according to two research approaches: one experimental and the other ethnographic. The experimental approach grounds on data collected during school experiments (university and secondary school) and analysed through some relevant theoretical lenses in Mathematics Education (e.g. Boundary Objects, as in [4]) to maximize the understanding of an unknown phenomenon by observing it from different standpoints. The ethnographic approach grounds on a three-year-long fieldwork conducted observing the Internet phenomenon in its natural habitat following an innovative methodology developed for this purpose. A data set of nearly 2000 memes has been collected and analysed through the theoretical framework used to investigate the Web 2.0 culture [3]. In the process of the research, the two approaches overlapped and intertwined, producing theoretical, methodological and empirical results that contribute in building a body of knowledge about exploring Internet phenomena with an educational purpose. Theoretically, mathematical memes are conceptualized as representations of mathematical statements with an epistemic power that nurtures mathematical discussions [2], and interpreted through a new semiotic tool that distinguishes levels of meanings. Methodically, an innovative use of ethnography framing multi-case focus studies is developed, that now can be used to explore Internet phenomena from the point of view of mathematics education. Empirically, the educational potentialities of mathematical memes are sampled. To sum up, the research shows that the contamination between mathematics and memes empowers both cultural realms: it upgrades the use of memes beyond their original subculture, and it expands the range of disciplinary signs traditionally pertaining to the domain of mathematics. These resources may contribute to fostering the cultural change aimed at bridging the discontinuity between out-of-school and in-school learning environments, as embraced by the recent literature about the future themes of mathematics education research [1].

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GLASSY BEHAVIOR IN MISMATCHED RANK-ONE MATRIX ESTIMATION

FRANCESCO CAMILLI

This contribution will focus on a topic that has recently attracted attention both from the Statistical Physics and Information Theory communities: the mismatched rank-one matrix estimation. In this setting the Statistician, who has to infer a rank-one matrix blurred by Gaussian additive noise, assumes a *prior* on its elements that does not match the real one. We will show that the problem can be mapped into a particular spin glass model whose free energy is identified by a variational principle on two order parameters: the Parisi overlap distribution and the Mattis magnetization. Once the free energy is given, the main Information Theoretic quantities, such as the Mean Square Error of the matrix estimation, are derived. The specific mismatch of a Bernoulli prior with the product of two Gaussians is analyzed in detail and shown to exhibit a rich behavior, including glassy phases.

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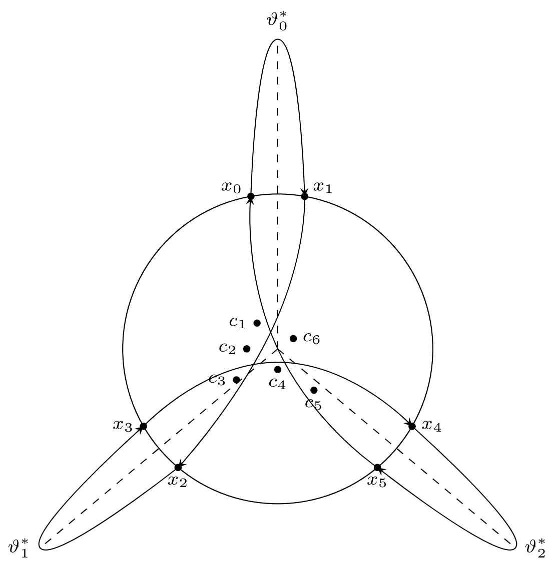
SYMBOLIC DYNAMICS FOR THE ANISOTROPIC N-CENTRE PROBLEM

GIAN MARCO CANNEORI

We consider the planar N-centre problem of Celestial Mechanics, assuming that each centre is endowed with a homogeneous anisotropic potential. As a peculiar difference with Newtonian potentials, the attraction field generated by every centre is no longer radial and the rotational symmetry is lost. These facts destroy the integrability of the system and prevent the use of collisions regularization methods such as the *Levi-Civita transform*. However, a law of conservation of the energy still holds and we show the existence of bounded periodic trajectories for *slightly* negative energies, employing both variational and perturbation methods. In particular, we show that, over a certain threshold on the homogeneity degree, all the solution trajectories of this system are collision-less. Consequently, the existence of a symbolic dynamics for the system is deduced, which is enriched by the important role played by the minimal central configurations of the potentials taken into account.

This is a joint work with Vivina Barutello and Susanna Terracini ([1]) .



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LUSIN APPROXIMATION THEOREMS OF ORDER m IN CARNOT GROUPS

MARCO CAPOLLI

Lusin's approximation theorem is a powerful tool used in analysis to approximate non-smooth maps. It asserts that, given a measurable map $f\colon \mathbb{R}^n \to \mathbb{R}$, there exists a continuous map $F\colon \mathbb{R}^n \to \mathbb{R}$ such that $f = F$ outside of a set that can be made arbitrarily small. Several refinements of Lusin's theorem show that more regular functions f can be approximated by functions F with higher regularity, such as C^m maps.

A large part of analysis in Euclidean spaces may be generalized to Carnot groups, i.e. Lie groups whose Lie algebra admits a stratification. The goal of this talk is to provide results analogous to Lusin's theorem for functions whose domain or target is one of those groups.

The first part is devoted to the presentation of [1], where we give sufficient conditions under which an absolutely continuous horizontal curve $\gamma : [a,b] \to \mathbb{H}^1$ can be approximated by a C^m horizontal curve. Here with $\mathbb{H}^1$ we mean the classical Heisenberg group, which is the simplest example of non-commutative Lie group. We also show that we cannot weaken the regularity assumptions, proving that our result is optimal in the context of the Heisenberg group.

In the second part of the presentation we focus our attention on [2]. Here the functions are real valued as in the classical result, however the domain is a general Carnot group. We prove that k-approximate differentiability almost everywhere is equivalent to admitting a Lusin approximation by $C^k_{\mathbb{G}}$ maps. We also prove that existence of an approximate $(k-1)$-Taylor polynomial almost everywhere is equivalent to admitting Lusin approximation by maps in a suitable Lipschitz function space.

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BOSE-EINSTEIN CONDENSATION FOR TWO DIMENSIONAL INTERACTING BOSONS IN THE GROSS-PITAEVSKII SCALING

CRISTINA CARACI

In this talk I will discuss a system of N bosons trapped in a two-dimensional box with area one, interacting through a repulsive potential with scattering length exponentially small in the number of particles, the so-called Gross-Pitaevskii regime. Assuming some regularity conditions on the interaction potential V, we show that low-energy states exhibit complete Bose-Einstein condensation, with almost optimal bounds on the number of orthogonal excitations.

I will also explain how it is possible, exploiting this result, to get further information on the low-lying excitation spectrum of the system. This is a joint work with S. Cenatiempo and B. Schlein, and part of my PhD thesis [1, 2, 3].

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KRYLOV SOLVABILITY IN A PERTURBATIVE FRAMEWORK

NOÈ ANGELO CARUSO

Solutions to abstract inverse linear problems $Af = g$ on Hilbert space $\mathcal{H}$ are often approximated by using finite linear combinations from the cyclic vector subspace generated by the operator A and datum g, also known as the *Krylov subspace* of A and g. These subspaces play an important role in applications: indeed, a Krylov-solvable inverse linear problem allows for the approximation of solution(s) by using very fast and well-known Krylov subspace methods.

This presentation is a first-round study of the phenomenon of Krylov-solvability of abstract inverse linear problems that are subject to small perturbations either in the operator A or the datum g. A particular focus is on monitoring phenomena related to the loss, gain, or preservation (i.e., the *stability*) of Krylov-solvability of inverse problems under perturbations. A meaningful way to describe the distance between Krylov subspaces that captures some of the most intuitive phenomena is also discussed. Throughout the presentation informative examples and results are given that unmask the complexity and range of phenomena encountered.

Though the presentation remains in the abstract, infinite-dimensional setting, perturbations of inverse linear problems are expected in applications for which one doesn't have precise information of the datum g and wishes to decide a-priori whether or not the problem is appropriately treatable using Krylov methods.

This is based on a series of joint works with Alessandro Michelangeli (Institute for Applied Mathematics, and Hausdorff Center for Mathematics, University of Bonn)

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A WASSERSTEIN INDEX OF DEPENDENCE FOR BAYESIAN NONPARAMETRIC MODELING

MARTA CATALANO

Optimal transport (OT) methods and Wasserstein distances are flourishing in many scientific fields as an effective means for comparing and connecting different random structures. In this talk we describe the first use of an OT distance between Lévy measures with infinite mass to solve a statistical problem. Complex phenomena often yield data from different but related sources, which are ideally suited to Bayesian modeling because of its inherent borrowing of information. In a nonparametric setting, this is regulated by the dependence between random measures: we derive a general Wasserstein index for a principled quantification of the dependence gaining insight into the models' deep structure. It also allows for an informed prior elicitation and provides a fair ground for model comparison. Our analysis unravels many key properties of the OT distance between Lévy measures, whose interest goes beyond Bayesian statistics, spanning to the theory of partial differential equations and of Lévy processes.
This is a joint work with Hugo Lavenant, Antonio Lijoi and Igor Prünster.

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EXCEPTIONAL SETS FOR HARDY SOBOLEV SPACES IN SEVERAL COMPLEX VARIABLES

NIKOLAOS CHALMOUKIS

The class of holomorphic Hardy Sobolev spaces in the unit ball of $\mathbb{C}^n$ is a family of spaces including the Hardy, Drury Arveson, Bergman and Dirichlet space.

In this talk we will focus on questions regarding exceptional sets both from function theoretic and a functional analytic perspective. These two approaches have led to the past in two different notions of exceptional sets. From the point of view of function theory, exceptional sets are sets where a function in the corresponding Hardy Sobolev space fails to have admissible limits and can be characterized as a null sets for some appropriately defined capacity [1]. While from the functional analysis perspective null sets, called totally null sets, play the role of Lebesgue measure zero sets in the Sz.-Nagy-Foias $H^\infty(\mathbb{D})$ functional calculus [2]. Our main theorem [3] proves the equivalence of the two notions. We will discuss also some interesting corollaries.

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CONTROLLING THE SPREAD OF INVASIVE BIOLOGICAL SPECIES

MARIA TERESA CHIRI

We consider a controlled reaction-diffusion equation, modeling the spreading of an invasive population. Our goal is to derive a simpler model, describing the controlled evolution of a contaminated set. We first analyze the optimal control of 1-dimensional traveling wave profiles. Using Stokes' formula, explicit solutions are obtained, which in some cases require measure-valued optimal controls. Then we introduce a family of optimization problems for a moving set and show how these can be derived from the original parabolic problems, by taking a sharp interface limit. In connection with moving sets, we show some results on controllability, existence of optimal strategies, and necessary conditions.

This is a joint work with Prof. Alberto Bressan and Dr. Najmeh Salehi (Penn State).

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VANISHING VISCOSITY IN MEAN-FIELD OPTIMAL CONTROL

GENNARO CIAMPA

We show the existence of Lipschitz-in-space optimal controls for a class of mean-field control problems with dynamics given by a non-local continuity equation. The proof relies on a vanishing viscosity method: we prove the convergence of the same problem where a diffusion term is added, with a small viscosity parameter.
By using stochastic optimal control, we first show the existence of a sequence of optimal controls for the problem with diffusion. We then build the optimizer of the original problem by letting the viscosity parameter go to zero.

This is a joint work with Francesco Rossi (University of Padova).

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IRREDUCIBLE GENERALIZED NUMERICAL SEMIGROUPS AND A GENERALIZATION OF WILF'S CONJECTURE

CARMELO CISTO

Let $\mathbb{N}$ be the set of non negative integers and d be a positive integer. A *generalized numerical semigroup*(GNS) is a submonoid S of $\mathbb{N}^d$ such that the set $\mathrm{H}(S) = \mathbb{N}^d \setminus S$ is finite. The set $\mathrm{H}(S)$ is called the set of *gaps* (or *holes*) of S, its cardinality $\mathrm{g}(S) = |\mathrm{H}(S)|$ is called the *genus* of S. Generalized numerical semigroups have been introduced in [5] as a straightforward generalization of the well known concept of numerical semigroup, that is a submonoid of $\mathbb{N}$ having finite complement in it. Some notions and results concerning numerical semigroups have been generalized to the context of GNSs in [3] and in other recent papers. One of the most important of these generalizations concerns with the concept of irreducibility. A GNS is said to be *irreducible* if it cannot be expressed as the intersection of two generalized numerical semigroups properly containing it. We prove that such GNSs are characterized by the set $\mathrm{SG}(S) = \{\mathbf{x} \in \mathrm{H}(S) \mid 2\mathbf{x} \in S, \mathbf{x}+\mathbf{s} \in S \text{ for all } \mathbf{s} \in S \setminus \{\mathbf{0}\}\}$, called the set of *special gaps* of S. We show in particular that S is irreducible if and only if $|\mathrm{SG}(S)| = 1$. This result is the starting point to obtain other properties and characterizations. For instance, if $S \subseteq \mathbb{N}^d$ is an irreducible GNS and $\mathrm{SG}(S) = \{\mathbf{f}\}$ then it is verified that $\mathbf{f}$ is the maximum in the set $\mathrm{H}(S)$ with respect to the natural partial order in $\mathbb{N}^d$. This fact allows to address a problem posed in [5], that is to define the analogue of the *Frobenius number* of a numerical semigroup for a GNS. Recalling that the Frobenius number of a numerical semigroup $S \subseteq \mathbb{N}$ is $\mathrm{F}(S) = \max(\mathrm{H}(S))$ (with the convention $\mathrm{F}(\mathbb{N}) = -1$), in [5] a *Frobenius element* of a GNS is defined up to a particular class of total orders in $\mathbb{N}^d$, called *relaxed monomial orders*. In particular, if $\prec$ is such an order on $\mathbb{N}^d$, the *Frobenius element* of a GNS $S \subseteq \mathbb{N}^d$ is defined as $\mathrm{F}_\prec(S) = \max_\prec(\mathrm{H}(S))$. Our results show that for irreducible generalized numerical semigroups it is verified that $\mathrm{F}_\prec(S) = \mathbf{f}$ for every relaxed monomial order $\prec$, where $\mathbf{f}$ is the unique element in $\mathrm{SG}(S)$. Another nice characterization for irreducible GNSs shows that a GNS $S \subseteq \mathbb{N}^d$ is irreducible if and only if there exists $\mathbf{f} = (f_1, \dots, f_d) \in \mathrm{H}(S)$ such that $\prod_{i=1}^d (f_i+1) = 2\,\mathrm{g}(S)$ or $\prod_{i=1}^d (f_i+1) = 2\,\mathrm{g}(S)-1$. Such equalities inspired us the formulation of a generalization in the context of GNSs of a well known conjecture for numerical semigroups, called *Wilf's conjecture* (see [4] for a survey on the argument). Such a conjecture is still an open (and intriguing) problem for numerical semigroups, even if it is known that it is true for a large number of classes of numerical semigroups. After defining our generalized version of the conjecture, we prove that it is true for all irreducible GNSs and some other classes of GNSs.

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CONSTRAINED OPTIMIZATION MODELS BASED ON UAV NETWORKS WITH 5G TECHNOLOGIES

GABRIELLA COLAJANNI

In recent years, the impact of 5G technology (the fifth-generation mobile network) is revolutionizing all social and economic sectors since, compared to previous networks, 5G has superior and better performance in terms of speed, latency, error, reliability, efficiency and so on and because 5G enables a new type of network designed to virtually connect everyone and everything together, including machines, objects and devices. Therefore, the purpose of this talk is to propose an optimization of a fleet of Unmanned Aerial Vehicles (UAVs), organized as a Flying Ad hoc Network (FANET) aimed at providing 5G network slices or on-demand services to users and devices on the ground. Moreover, the FANET, which is emerging as an alternative access technology for regions that do not have fixed infrastructures or are difficult to reach (perhaps after a disastrous event), is represented by a set of UAVs that communicate with each other even in rural or post-disaster areas, often not even covered by the electricity grid or in marine environments. The determined three-level Networks are able to represent all the fundamental elements of different structures of 5G network and slice architectures. The proposed nonlinear network-based multilevel optimization models for the provision of 5G services are very useful for problem solving and to determine the optimal distributions flows. The associated Variational inequality formulations are proposed, and some numerical examples are presented to validate the effectiveness of the models.

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REFINED GAUSS–GREEN FORMULAS

GIOVANNI E. COMI

The Gauss–Green and integration by parts formulas are of significant relevance in many areas of analysis and mathematical physics. Their importance motivated several investigations to obtain extensions to more general classes of integration domains and weakly differentiable vector fields, thus ultimately leading to the definition of divergence-measure fields. These are L^p-summable vector fields on $\mathbb{R}^n$ whose distributional divergence is a Radon measure. It is not difficult to notice that the divergence-measure fields are a generalization of the vector fields of bounded variation.

I shall present the approach to the theory of divergence-measure fields in the Euclidean framework developed in [2], a joint work with Kevin R. Payne. We consider the case $p = \infty$ and prove that the Gauss–Green formulas hold on sets of finite perimeter by suitably manipulating the Leibniz rule for essentially bounded divergence-measure fields and essentially bounded scalar functions of bounded variation. In this way, we define the interior and exterior normal traces of divergence-measure fields and show that they are essentially bounded functions on the reduced boundary of the integration set.

Due to the robustness of the Euclidean theory of divergence-measure fields, we can extend it to some non-Euclidean context. In particular, we develop a theory of divergence-measure fields in noncommutative stratified nilpotent Lie groups, based on [1], a joint work with Valentino Magnani. Since the Euclidean theory relies heavily on De Giorgi's blow-up theorem and related fine properties of functions of bounded variation, which do not hold in general stratified groups, we provide alternative approximation arguments in order to prove a Leibniz rule for essentially bounded horizontal divergence-measure fields and essentially bounded scalar function of bounded h-variation. As a consequence, we achieve the existence of normal traces and the Gauss–Green theorem on sets of finite h-perimeter. Then, we consider some particular cases; that is, fields whose divergence is an absolutely continuous measure and integration over sets with Euclidean finite perimeter.

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SURFACES CLOSE TO THE SEVERI LINES

FEDERICO CONTI

We are interested in the study of surfaces of general type with maximal Albanese dimension for which the quantity $K_X^2 - 4\chi(\mathcal{O}_X) - 4(q(X) - 2)$ vanishes or is "small" (where $q(X)$ denotes the dimension of the Albanese variety of X). This is related to the Severi inequality which states that such a surfaces satisfies $K_X^2 \geq 4\chi(\mathcal{O}_X)$ (cf. [7]) and equality occurs if and only if the canonical model of X is a double cover of its Albanese variety ramified over an ample divisor (cf. [1]). It is then natural to ask if there is a stronger inequality in which also the irregularity $q(X)$ is involved and this has been achieved by Lu and Zuo ([6]) who have proved that a surface X as above for which $K_X^2 < \frac{9}{2}\chi(\mathcal{O}_X)$ holds, satisfies $K_X^2 \geq 4\chi(\mathcal{O}_X) + 4(q(X) - 2)$.

We show, refining the result of Lu and Zuo, that such a surface with $q(X) \geq 3$ satisfies $K_X^2 = 4\chi(\mathcal{O}_X) + 4(q(X) - 2)$ if and only if its canonical model is a double cover of a product elliptic surface $C \times E$ ramified over an ample divisor $R \sim_{lin} C_1 + C_2 + \sum_{i=1}^{2} dE_i$ with at most negligible singularities. Moreover, if $K_X^2 > 4\chi(\mathcal{O}_X) + 4(q(X) - 2)$ then $K_X^2 \geq 4\chi(\mathcal{O}_X) + 8(q(X) - 2)$ and equality occurs if and only if the canonical model of X is a double cover of an elliptic surface Y without singular fibres ramified over an ample divisor R with at most negligible singularities for which $K_Y.R = 8(q - 2)$.

Because these results are intimately related to the theory of double covers, it is not difficult to see that they hold over any algebraically closed field of characteristic different from 2. Nonetheless, it is possible to prove the same result in characteristic 2 adding some ad hoc hypothesis (e.g. the Albanese morphism is separable).

Some of the results here involved, such as the Severi inequality, have been extended to higher dimensional variety ([3] and [4]): it is then natural to ask, for a possible future work, if it is possible to extend these results for varieties of general type with maximal Albanese dimension of dimension greater than or equal to 3.

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HUMAN–INDUCED OSCILLATIONS IN A NETWORK LANDSCAPE MODEL

ROSSELLA DELLA MARCA

The concept of territorial resilience is closely associated with the ability of a local system to tolerate the impact caused by adverse circumstances. Here, we investigate the role of the human factor in the resilience of environmental systems. To this end, a coupled human–landscape model is proposed for a network of Landscape Units (LUs), where each LU is endowed by a system of ODEs for the time evolution of two environmental variables (fraction of green areas and production of bio–energy) and one human variable (fraction of environmentalists in the population). Injunctive social norms that tend to population conformity are taken into account. First the dynamics in each LU is analytically investigated, with reference to equilibria and their stability; the possible occurrence of Hopf bifurcations is proved, with consequent periodic oscillations of environmental and human variables, as typical of resilient territories. The numerical investigation shows that such oscillations may disappear by global heteroclinic bifurcations. Then, the connectivity between the LUs is considered, with the aim of pointing out the effects of the single LU dynamics on the network landscape model. Numerical simulations of different scenarios are performed in a sample model of an environmental system in Northern Italy.

This is a joint work [2] with Maria Groppi and Ana Jacinta Soares.

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PROBABILISTIC VS DETERMINISTIC GAMBLERS

VALENTINO DELLE ROSE

This is a joint work with Laurent Bienvenu and Tomasz Steifer.

Is it possible for a gambler using a probabilistic betting strategy to become arbitrarily rich when all gamblers betting according to a deterministic strategy earn only a bounded capital?

We investigate this question in the context of algorithmic randomness, introducing the new notion of *almost everywhere computable randomness*.

Algorithmic randomness aims at formalizing the intuitive concept of randomness for single outcomes, which are usually modelled as infinite binary sequences. A popular way to do so is via the unpredictability approach. We fix a certain class $\mathcal{C}$ of effective gambling strategies for the following game. The bits of an infinite sequence X are revealed in ascending order. When the strategy $B \in \mathcal{C}$ has already seen n many bits of X, B bets a certain amount α of its capital that the $n+1$-th bit of X is, say, 0: if B is right, then B wins α, otherwise B loses α. We say that the strategy B *succeeds* on X if its capital tends to infinity throughout the above infinite game, and we consider a sequence X as *random* (with respect to the given class $\mathcal{C}$) if no betting strategy in $\mathcal{C}$ succeeds on X. In particular, we talk of *computable randomness* when we consider only total computable betting strategies, and of *partial computable randomness* if we also allow partial computable ones. In both cases, however, these strategies are deterministic.

In our framework, instead, we also consider probabilistically effective betting strategies: intuitively speaking, we consider effective betting strategies which, in addition, are allowed to flip a fair coin before placing their bet (and possibly betting accordingly). More formally, we assume that the infinite sequence Y of coin tosses has been drawn in advance and given as an oracle to a partial computable betting strategy B (thus obtaining a partial Y-computable betting strategy which we denote by B^Y): hence, we say that a sequence X is *almost everywhere computably random* if, for any partial computable betting strategy B, we have that

$$\mu\left(\left\{Y : \ B^Y \text{ is total and succeeds on } X\right\}\right) = 0,$$

where μ denotes the uniform measure on the space of infinite binary sequences.

We show that probabilistic betting strategies are in fact stronger than deterministic ones, by building a partial computable random sequence which is not almost everywhere computably random. It is worth noticing that this is an unusual and unexpected result in computability theory, because of a classical theorem stating that every sequence which can be computed by a probabilistic algorithm with positive probability is in fact deterministically computable ([1]).

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SPARSE RECOVERY VIA FAST NONNEGATIVE LEAST SQUARES

MONICA DESSOLE

We address the problem of finding the sparsest solution to an underdetermined system of linear equations, i.e. the solution with most zero entries. This problem, with many applications in the field of signal processing, is known as sparse recovery and its solution is NP-hard in general, although it is well known that ℓ_1-minimization leads the sparsest solution for a restricted class of matrices [1]. Similarly, it has been observed (see e.g. [3]) that the nonnegativity constraint naturally enhances sparsity of the solution, leading to solve a NonNegative Least Squares (NNLS) problem. Many NNLS solvers require the objective function to be strictly convex, which is not true when dealing with underdetermined linear systems. The Lawson-Hanson algorithm [4] does not suffer from this drawback, however it is mainly based on BLAS2 operations, yielding limited performance. We propose to employ a recent block column selection strategy [2], devising a new algorithm we called Lawson-Hanson with Deviation Maximization (LHDM) that allows to exploit BLAS3 operations, leading to better performance. Numerical results are presented with an extensive campaign of experiments, where we compare LHDM against several ℓ_1-minimization solvers, showing that it is a valuable alternative for sparse recovery in terms of both quality of the solution found and execution times.

This is a joint work with Prof. Fabio Marcuzzi.

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ENERGETIC BOUNDARY ELEMENT METHOD FOR 2D ELASTODYNAMICS PROBLEMS IN TIME DOMAIN

GIULIA DI CREDICO

In this presentation I will provide an overview on the theoretical and numerical aspects involved in the application of the Energetic Boundary Element Method (Energetic BEM) to the resolution of 2D elastodynamic problems in exterior unbounded domains [1]. This method implies the reformulation of the related differential problem in terms of *Boundary Integral Equations* (BIEs), which depend on characteristic integral operators and are suitable to solve problems equipped by Dirichlet or Neumann datum at the boundary. A space-time approach is considered, then the BIEs are reformulated in a weak form, based on energy arguments, and numerically solved by means of the Energetic BEM. This approach ensures a good theoretical setting in case of single layer weak formulations, leading to an accurate and stable in time resolution of the BIEs and to a precise approximation of the external unknown displacement. Several numerical tests will be therefore presented and discussed: the correctness of the Energetic BEM implementation is in particular proved showing the convergence in time of solutions of specific indirect BIEs towards analytical elastostatic functions, these latter calculated on the base of the datum imposed on a straight open obstacle. With other numerical results we moreover aim to study the decay of the energetic approximation errors obtained with the use of *hp* and *graded meshes*, useful to refine the boundary of a polygonal domain towards critical corner points where the BIEs solutions are featured by a singular behaviour [4].
The presentation will also take into consideration the numerical aspects entailed by the discretization of the energetic weak formulations, for which solving the integral problem becomes equivalent to the resolution of a linear system, where the block Toeplitz structured matrix has entries represented by quadruple space-time integrals: the accurate numerical computation of these elements is a crucial phase for the Energetic BEM implementation, especially because of the peculiar spatial singularities characterizing the related integral kernel [2, 3].
In the end I will discuss a strategy to reduce the computational costs of the Energetic BEM in case of large scale acoustic and elastodynamic wave propagation problems [5]. Thanks to a compression technique based on the *Adaptive Cross Approximation* (ACA), I will show a remarkable reduction in the assembly time of the linear system and in the memory requirements for the storage of the Toeplitz matrix, without altering the precision of the numerical results.

This is a joint work with Alessandra Aimi, Mauro Diligenti, Chiara Guardasoni [2, 3], Heiko Gimperlein, Ernst P. Stephan [4] and Luca Desiderio [5].

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STRUCTURE-PRESERVING DISCRETIZATIONS TO STOCHASTIC DIFFERENTIAL EQUATIONS

STEFANO DI GIOVACCHINO

In this talk, we focus our attention on structure-preserving numerical discretizations of selected stochastic differential equations (SDEs). The first part of this talk is devoted to the analysis of stochastic Runge-Kutta and ϑ-methods applied to mean-square dissipative SDEs [6], showing the mean-square contractive character of such discretizations, under suitable stepsize restrictions or algebraic constraints on the coefficients of the method [4, 5]. The second part of this talk investigates stochastic Hamiltonian systems (both of Itô and Stratonovich type [1, 2, 7]) under time discretizations. In particular, we present long-term estimates for the numerical Hamiltonian, highlighting the eventual conservative nature of the underlying numerical method [3]. Selected numerical results are provided to confirm the effectiveness of the theoretical analysis.

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EXTERIOR ALGEBRA: NEW AND OLD RESULTS, OPEN PROBLEMS AND GENERALIZATIONS

SABINO DI TRANI

Let $\mathfrak{g}$ be a simple Lie algebra over $\mathbb{C}$. The adjoint action of $\mathfrak{g}$ on itself induces an action of $\mathfrak{g}$ on $\Lambda\mathfrak{g}$ and $S(\mathfrak{g})$, the exterior algebra and the symmetric algebra over $\mathfrak{g}$ respectively.
Some interesting questions about their irreducible decompositions arise naturally:

Q1: Which are the irreducible representations appearing in $S(\mathfrak{g})$ and in $\Lambda\mathfrak{g}$?

Q2: Is it possible to find some formulae for their (graded) multiplicities?

Concerning the symmetric algebra it is proved that the multiplicities are encoded by some special Kazhdan-Luzstig polynomials, determined by the affinization of the Weyl group of $\mathfrak{g}$.
On the other side, despite of the finite dimension (as vector space) of $\Lambda\mathfrak{g}$, determining the irreducible components in the exterior algebra seems to be quite difficult.
Kostant proved (see [6]) that $\Lambda\mathfrak{g}$ is isomorphic, as $\mathfrak{g}$ module, to the direct sum of $2^{\mathrm{rk}\mathfrak{g}}$ copies of the tensor product representation $V_\rho \otimes V_\rho$, where ρ denotes the Weyl vector of $\mathfrak{g}$, with respect to the choice of a set of positive roots in the root system Φ of $\mathfrak{g}$. Unfortunately, this decomposition seems not to be compatible with the grading and links the problem of finding the representations appearing in $\Lambda\mathfrak{g}$ to a Clebsch-Gordan decomposition problem, that is generally hard to be solved. A description of irreducibles in $\Lambda\mathfrak{g}$ it is currently known only in type A and is conjectural for other classical cases (a complete exposition on this topic is contained in [1]).
In the talk I give an overview of some open conjectures and many elegant results proved in the last Century, focusing on graded multiplicities in $\Lambda\mathfrak{g}$ of certain class of irreducible representations. The most famous results in this direction is a theorem due to Hopf, Koszul and Samelson that describes the subalgebra of invariants in $\Lambda\mathfrak{g}$ as an exterior algebra over a suitable set of homogeneous generators of prescribed degrees. Many results on this topic are contained in [7], where Reeder conjectured that it is possible to compute the graded multiplicities of a special class of representations, called "small", reducing to a "Weyl group representations" problem.
We outline the strategy we used to prove Reeder's Conjecture in the classical cases (see [4] and [5]) and we show how our formulae can be rearranged involving the generalized exponents, obtaining a generalization of some well known expressions for graded multiplicities of the adjoint and little adjoint representations in $\Lambda\mathfrak{g}$. As a consequence, we conjecture a generalization of the results contained [3] and [2] to a larger family of special covariants in the exterior algebra.

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LINEAR STABILITY PROPERTIES OF SHEAR FLOWS IN INHOMOGENEOUS FLUIDS

MICHELE DOLCE

A fundamental question in hydrodynamics is the understanding of stability properties of shear flows, with the earlier studies of Reynolds, Rayleigh and Kelvin, among many others, dating back to the end of the nineteen century. In the last decade, especially for homogeneous fluids, the problem received renewed attention thanks to the introduction of new analytical techniques particularly useful to tackle these classical problems, e.g. [2]. When the fluid is inhomogeneous, the dynamics is richer and less is known. In this talk, I will present two results obtained in my PhD thesis [4]. I will first discuss quantitative linear stability properties of the Couette flow with constant density in an isentropic compressible fluid [1]. Then, I will show some properties of a class of linearly stratified shear flows in an inhomogeneous fluid under the Boussinesq approximation [3].

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ENTROPY MARTINGALE OPTIMAL TRANSPORT AND NONLINEAR PRICING-HEDGING DUALITY

ALESSANDRO DOLDI

The aim of this paper is to develop a duality between a novel Entropy Martingale Optimal Transport problem (A) and an associated optimization problem (B):

$$(A) := \inf_{Q \in \mathrm{Mart}(\Omega)} \left(E_Q\left[c\right] + \mathcal{D}_U(Q)\right) = \sup_{\Delta \in \mathcal{H}} \sup_{\varphi \in \mathbf{\Phi}_\Delta(c)} S^U(\varphi) =: (B). \tag{1}$$

In (A) we follow the approach taken in the Entropy Optimal Transport (EOT) primal problem by Liero et al. [2] but we add the constraint, typical of the Martingale Optimal Transport (MOT) theory in [1], that the infimum of the cost functional is taken over martingale probability measures, instead of finite positive measures, as in Liero et al. The Problem (A) differs from the corresponding problem in Liero et al. not only by the martingale constraint, but also because we admit less restrictive penalization terms $\mathcal{D}_U$, which may not have a divergence formulation.

In Problem (B), the outer supremum is taken over admissible integrands (Δ, given by continuous functions). Every such integrand models a self-financing trading strategy, with terminal wealth given by the corresponding stochastic integral I^Δ. In the inner supremum of (B), the class Φ_Δ consists of static parts of semistatic subhedging strategies for the contingent claim c involving the strategy Δ, namely $\mathbf{\Phi}_\Delta(c) = \{\varphi = [\varphi_0, \ldots, \varphi_T] \mid \sum_{t=0}^T \varphi_t(x) + I^\Delta(x) \le c(x) \ \ \forall x \in \Omega\}$. The objective functional S^U, associated via Fenchel conjugacy to the term $\mathcal{D}_U$, is not any more linear, as in OT or in MOT. This leads to a novel optimization problem which also has a clear financial interpretation as a nonlinear subhedging value.

Mathematically speaking, the duality we develop is a relaxed version of the classical MOT problem. MOT problems have been widely studied in Mathematical Finance due to the fact that they yield robust bounds for sub/superhedging prices when options can be bought and sold in the market. Historically, it is assumed that the marginals of the pricing measures are known, since these can be inferred from market data. Since in practice these marginals can only be known in an approximate form due to limited data availability, it is also financially relevant to study MOT problems in which marginals are not expected to exactly match the candidate, data-inferred ones. This is among the aims of this work.

Our theory allows us to establish a nonlinear robust pricing-hedging duality, which covers a wide range of known robust results. We also focus on Wasserstein-induced penalizations and we study how the duality is affected by variations in the penalty terms, with a special focus on the convergence of EMOT to the extreme case of MOT.

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SURFACES WITH CANONICAL MAP OF HIGH DEGREE

FEDERICO FALLUCCA

Let S be a compact complex surface. If the image of the canonical map ϕ is a surface, then we can consider its *degree* d. Beauville obtained in [1] that $d \leq 9 + \frac{27-9q}{p_g-2}$, where $q = h^{1,0}(S)$ and $p_g = h^{2,0}(S)$ are the Hodge numbers of S. As noted first by Persson, the maximum possible degree is 36 and if $d > 27$ then $q = 0$ and $p_g = 3$.

A question posed by M. Lopes and R. Pardini in [2] is if for each $d \leq 36$ there exists an algebraic surface S such that the degree of its canonical map is equal to d. At the moment there are only examples in literature of surfaces with canonical map of degree $d = 2, \cdots, 9, 12, 16, 20, 24, 27, 32, 36$.

I consider the so called regular product-quotient surfaces, surfaces birational to a quotient $(C_1 \times C_2)/G$, where C_i are curves and G is a finite group acting separately on both factors. The aforementioned results suggest to produce systematically examples of smooth regular product-quotient surfaces with $q = 0$ and $p_g = 3$ following the techniques in [3] and [4] and then to study their canonical maps. There is no general way to compute the canonical map of product-quotient surfaces. However the assumption $p_g = 3$ implies $d = M^2$, where M is the mobile part of the canonical system of the blow-up of S in the base locus of its canonical system. In this case to compute d we have only to describe the intersection of three canonical divisors generating the canonical system $|K_S|$. In this direction I proved the following

Theorem 1. *Let C be a curve, $G < Aut(C)$ be a finite group such that $C/G \cong \mathbb{P}^1$ and let $\pi : C \to \mathbb{P}^1$ be the quotient map. Let $\chi \in Irr(G)$ be an irreducible character of G, call ρ_χ its irreducible representation. Denote by $H^{1,0}(C)^\chi$ the corresponding isotypic component of the induced representation of G on $H^{1,0}(C)$. Call $|K_C|^\chi$ the associated subsystem of the canonical linear system of C given by the isotypic component $H^{1,0}(C)^\chi$. Then the base locus of $|K_C|^\chi$ is*

$$Bs(|K_C|^\chi) = \sum_{q \in Crit(\pi)} (m_q - a_q^\chi - 1)\pi^{-1}(q) \tag{1}$$

where h is the local monodromy of a point $p \in \pi^{-1}(q)$, $m_q := o(h)$ and a_q^χ is defined as

$$a_q^\chi := \max\{\lambda \in [0, \cdots, m_q - 1] : e^{\frac{2\pi i}{o(h)} \cdot \lambda} \ \textit{is an eigenvalue of} \ \rho_\chi(h)\}$$

In this talk I will explain how this theorem can be used to investigate the base locus of the canonical system of a product-quotient surface. As application I will give examples of algebraic surfaces with new canonical degrees, 10, 11, 14 and 18, together with two new examples of product-quotient surfaces with $d = 24, 32$.

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SOME REGULARITY RESULTS FOR THE 3D EVOLUTION NAVIER-STOKES EQUATIONS UNDER NAVIER BOUNDARY CONDITIONS IN SOME LIPSCHITZ DOMAINS

ALESSIO FALOCCHI

For the evolution Navier-Stokes equations in bounded 3D domains, it is well-known that the uniqueness of a solution is related to the existence of a regular solution. They may be obtained under suitable assumptions on the data and smoothness assumptions on the domain (at least $C^{2,1}$). With a symmetrization technique, we prove these results in the case of Navier boundary conditions in a wide class of merely *Lipschitz domains* of physical interest, that we call *sectors*, see Figure 1. The validity/failure of a suitable Poincaré-type inequality is also discussed to complete the proof. This is a joint work with Filippo Gazzola, Politecnico di Milano.

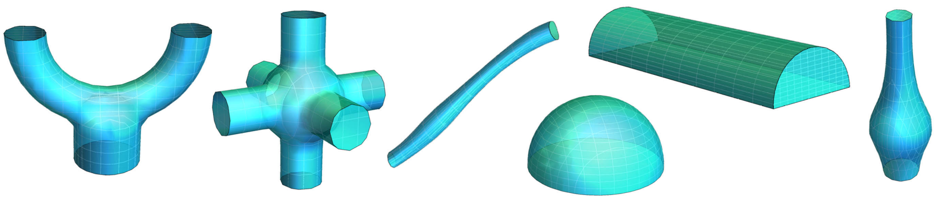

FIGURE 1. Some examples of *sectors*.

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INFINITESIMAL DEFORMATIONS AND THE EXTENDED TROPICAL VERTEX GROUP

VERONICA FANTINI

I will discuss the relationship between scattering diagrams and infinitesimal deformations of holomorphic pairs, which Fukaya outlined in [4], and which I studied in my PhD thesis [3].

Scattering diagrams were introduced by Kontsevich and Soibelman in the context of mirror symmetry [7]. They are defined algebraically, in terms of pro-nilpotent Lie groups, but in many applications they have a combinatorial structure which encodes enumerative geometric data (as in [5], [8], [6], [1], [3], for example).

I will cosider a holomorphic pair $(\check{X}, \check{E})$ where $\check{X}$ is the total space of a torus fibration and $\check{E} \to \check{X}$ is an holomorphic vector bundle. My main construction builds on a recent work of Chan, Conan Leung, and Ma [9], where the authors study the relationship between scatteirng diagrams and infinitesimal deformations of $\check{X}$. The new feature introduced in [2] is the extended tropical vertex group $\tilde{\mathbb{V}}$ where the scattering diagrams are defined. It consists of the *large volume limit asymptotics* of the automorphisms that act on deformations of $(\check{X}, \check{E})$. The asymptotics are taken in the limit $\hbar \to 0$, where $\hbar$ is a formal parameter that rescales the complex structure of $\check{X}$. I will sketch the proof of one of the results from [2]:

Theorem 1. *Let $\mathfrak{D}$ be a scattering diagram in $\tilde{\mathbb{V}}$.*

- *One can define an associated 1-form*
$$\Phi_{\mathfrak{D}} = a_1(\hbar)\, t + a_2(\hbar)\, t^2 + a_3(\hbar)\, t^3 + \dots,$$
which is a formal series in t with coefficients in $\Omega^1(\check{X}, \mathrm{T}^{1,0}\check{X} \oplus \mathrm{End}\, \check{E})$.

Suppose $\mathfrak{D}$ consists of two non-parallel walls. Then:

- *The Maurer-Cartan equation which governs deformations of $(\check{X}, \check{E})$ has a unique solution Φ which matches $\Phi_{\mathfrak{D}}$ at first order in t.*
- *The asymptotics of Φ at $\hbar \to 0$ give a saturation of $\mathfrak{D}$.*

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CAPACITARY POTENTIALS IN RIEMANNIAN MANIFOLDS AND GEOMETRIC APPLICATIONS

MATTIA FOGAGNOLO

In this talk I will discuss, in manifolds (M, g) with nonnegative Ricci curvature, monotonicity formulas for suitable integral quantities defined along the level sets of the p-capacitary potential of a bounded $\Omega \subset M$ with smooth boundary. Various analytic/geometric consequences are derived.

The most general purely geometric inequality we obtain is given by the Minkowski Inequality

$$\left(\frac{|\partial\Omega|}{|\mathbb{S}^{n-1}|}\right)^{\frac{n-2}{n-1}} \mathrm{AVR}(g)^{\frac{1}{n-1}} \leq \frac{1}{|\mathbb{S}^{n-1}|} \int\limits_{\partial\Omega} \left|\frac{H}{n-1}\right| d\sigma, \tag{1}$$

for *outward minimizing* domains $\Omega \subset M$, where H is the mean curvature of $\partial\Omega$ and $\mathrm{AVR}(g)$ is the asymptotic volume ratio of (M, g).

Moreover we show that equality holds true if and only if $(M \setminus \Omega, g)$ is isometric to a truncated cone over $\partial\Omega$.

The arguments and the results involve many other important concepts such as isoperimetric/isocapacitary inequalities, outward minimising sets and the Inverse Mean Curvature Flow, that will be briefly discussed.

The talk is mainly based on the papers [1], [2], [3].

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A RECENT PERTURBATIVE METHOD TO THE FREE BOUNDARY REGULARITY IN THE ONE-PHASE STEFAN PROBLEM

NICOLÒ FORCILLO

In this talk, I will focus on the free boundary regularity in the one-phase Stefan problem

$$\begin{cases} u_t = \Delta u & \text{in } (\Omega \times (0,T]) \cap \{u > 0\}, \\ u_t = |\nabla u|^2 & \text{on } (\Omega \times (0,T]) \cap \partial\{u > 0\}, \end{cases} \tag{1}$$

with $\Omega \subset \mathbb{R}^n$, $u : \Omega \times [0,T] \to \mathbb{R}$, $u \geq 0$. Specifically, I will present a recent approach to the study of the regularity for flat free boundaries of such problem, developed in [7], a joint work with D. De Silva and O. Savin.

In general, in Stefan type problems, free boundaries may not regularize instantaneously. In particular, there exist examples in which Lipschitz free boundaries preserve corners, see for instance [5]. However, in the two-phase Stefan problem, Athanasopoulos, Caffarelli and Salsa showed in [1] that Lipschitz free boundaries in space-time become smooth under a nondegeneracy condition. Moreover, they established the same conclusion in [2] for sufficiently "flat" free boundaries. Their techniques are based on the original work of Caffarelli in the elliptic case [3, 4].

The main result in [7] is essentially equivalent to the previously mention flatness result in [2]. Nevertheless, the method in [7] takes inspiration from the elliptic counterpart established by D. De Silva in [6]. The approach in [7] relies on perturbation arguments leading to a linearization of the problem. In this talk, I will discuss the main steps of such approach, focusing on the main ideas. In particular, I will prove the following result, see [7].

Theorem 1. *Fix a constant K (large) and let u be a solution to the one-phase Stefan problem* (1) *in $B_\lambda \times [-K^{-1}\lambda, 0]$ for some $\lambda \leq 1$. Assume that*

$$|u| \leq K\lambda, \qquad u(x_0, t) \geq K^{-1}\lambda \quad \textit{for some} \quad x_0 \in B_{\frac{3}{4}\lambda}.$$

There exists ε_0 depending only on K and n such that if, for each t, $\partial_x\{u(\cdot, t) > 0\}$ is ε_0-flat in B_λ, then the free boundary $\partial\{u > 0\}$ (and u up to the free boundary) is smooth in $B_{\frac{\lambda}{2}} \times [-(2K)^{-1}\lambda, 0]$.

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EQUIVARIANT BIRATIONAL TRANSFORMATIONS AND GENERALIZATIONS OF MATRIX INVERSION

ALBERTO FRANCESCHINI

This is joint work with my supervisor Prof. Solá Conde.

Birational transformations are strongly linked with torus actions, see [4] and the references therein. Going back to the works of Thaddeus [7, 8] and WłWlodarczyk [9], given a birational map ψ between complex projective varieties, there exists an algebraic variety X admitting a $\mathbb{C}^*$-action such that ψ is the map induced among two GIT-quotients of X by $\mathbb{C}^*$.

On the other hand, as pointed out in [3], if we start with a smooth projective variety X admitting a non-trivial $\mathbb{C}^*$-action, then the induced birational transformation ψ between the GIT-quotients can be read out by the properties of the $\mathbb{C}^*$-invariants curves and the Theorem of Białynicki-Birula [1, 2].

We study the birational transformations associated with equalized $\mathbb{C}^*$-actions on a rational homogeneous variety with Picard number one, extending some results of [5].

Theorem 1. *Let G be a simple algebraic group and $X = G/P$ be a rational homogeneous variety of Picard number one. Consider the equalized $\mathbb{C}^*$-action on X determined by a short grading*

$$\mathfrak{g} = \mathfrak{g}_- \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_+$$

of the Lie algebra of G. Let w_0 be a representative in the class of the longest element in the Weyl group of $\mathfrak{g}$. Suppose also that the action is balanced, i.e. $t^{-1} = w_0 t w_0^{-1}$ for all $t \in \mathbb{C}^*$.

Following [2], let Y_-, Y_+ be sink and source of the action. Then the induced birational map

$$\psi : \mathbb{P}_{Y_-}\left(\mathcal{N}_{Y_-|X}\right) \dashrightarrow \mathbb{P}_{Y_+}\left(\mathcal{N}_{Y_+|X}\right)$$

between the projectivization of normal bundles is completely determined by the inversion map $j : \mathfrak{g}_- \dashrightarrow \mathfrak{g}_-$ of a Jordan algebra as in [6]. Furthermore, if the Lie algebra of P is conjugate to $\mathfrak{g}_- \oplus \mathfrak{g}_0$, then ψ is obtained as the composition of the projectivization of j with the linear isomorphism $\mathrm{Ad}_{w_0} : \mathbb{P}(\mathfrak{g}_-) \longrightarrow \mathbb{P}(\mathfrak{g}_+)$.

We also have a theorem that describes ψ explicitly when $\mathfrak{g}$ is a simple Lie algebra of classic type, which the space in this abstract is too narrow to contain.

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NEURAL ARCHITECTURE SEARCH VIA STANDARD MACHINE LEARNING METHODOLOGIES

GIORGIA FRANCHINI

In the context of deep learning, the more expensive computational phase is the full training of the learning methodology. Indeed, its effectiveness depends on the choice of proper values for the so-called hyperparameters, namely the parameters that are not trained during the learning process, and such a selection typically requires an extensive numerical investigation with the execution of a significant number of experimental trials. The aim of this work is to investigate how to choose the hyperparameters related to both the architecture of a Convolutional Neural Network (CNN), such as the number of filters and the kernel size at each convolutional layer, and the optimisation algorithm employed to train the CNN itself, such as the steplength, the mini-batch size and the potential adoption of variance reduction techniques. The main contribution of this work consists in introducing an automatic Machine Learning technique to set these hyperparameters in such a way that a measure of the CNN performance can be optimised [1]. In particular, given a set of values for the hyperparameters, we propose a low-cost strategy to predict the performance of the corresponding CNN, based on its behavior after only few steps of the training process. To achieve this goal, we generate a dataset whose input samples are provided by a limited number of hyperparameter configurations together with the corresponding CNN measures of performance obtained with only few steps of the CNN training process, while the label of each input sample is the performance corresponding to a complete training of the CNN.

Such dataset is used as training set for a Support Vector Machines for Regression and/or Random Forest techniques to predict the performance of the considered learning methodology, given its performance at the initial iterations of its learning process. Furthermore, by a probabilistic exploration of the hyperparameter space, we are able to find, at a quite low cost, the setting of a CNN hyperparameters which provides the optimal performance. The results of an extensive numerical experimentation, carried out on CNNs, together with the use of our performance predictor with NAS-Bench-101 [2], highlight how the proposed methodology for the hyperparameter setting appears very promising.

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MULTIPLE CONSTANT SIGN AND NODAL SOLUTIONS FOR THE FRACTIONAL p-LAPLACIAN

SILVIA FRASSU

In this talk we study a pseudo-differential equation driven by the degenerate fractional p-Laplacian, under Dirichlet type conditions in a smooth domain. First we show that the solution set within the order interval given by a sub-supersolution pair is nonempty, directed, and compact in a suitable fractional Sobolev space, hence endowed with extremal elements. Then, assuming that the reaction term is $(p-1)$-sublinear at infinity and asymptotically linear near the origin without resonance on the first eigenvalue, we prove the existence of a smallest positive and a biggest negative solution, combining variational methods with truncation techniques. Finally, under more restrictive assumptions on the behavior of the reaction term near the origin, we will show the existence of a nodal solution between the smallest positive and biggest negative solution.

This is a joint work with Antonio Iannizzotto.

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DEEP LEARNING-BASED REDUCED ORDER MODELS FOR NONLINEAR PARAMETRIZED PDES: APPLICATION TO CARDIAC ELECTROPHYSIOLOGY

STEFANIA FRESCA

Conventional reduced order models (ROMs) anchored to the assumption of modal linear superimposition, such as proper orthogonal decomposition (POD), may reveal inefficient when dealing with nonlinear time-dependent parametrized PDEs, especially for problems featuring coherent structures propagating over time, such as cardiac electrophysiology (EP). To enhance ROM efficiency, we propose a nonlinear approach to set ROMs by exploiting deep learning (DL) algorithms as convolutional neural networks. In the resulting DL-ROM, both the nonlinear trial manifold and the nonlinear reduced dynamics are learned in a non-intrusive way by relying on DL algorithms trained on a set of full order model (FOM) snapshots, obtained for different parameter values. Performing then a former dimensionality reduction on FOM snapshots through POD and using a suitable multi-fidelity pretraining enable, when dealing with large-scale FOMs, to speed-up training times, and decrease the network complexity, substantially. Accuracy and efficiency of the DL-ROM technique are assessed on different parametrized PDE problems in cardiac EP, representing both physiological and pathological scenarios, computational mechanics and fluid dynamics, where new queries to the DL-ROM can be computed in real-time. In particular, DL-ROMs are shown to be able to solve, after the training stage, cardiac EP problems on realistic geometries, for any new scenario in real-time, even in extremely challenging contexts such as re-entry and re-entry break-up problems, modeling the triggering phenomena related with cardiac arrhythmias.

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NONDEGENERATE ABNORMALITY AND GAP PHENOMENA IN OPTIMAL CONTROL WITH STATE CONSTRAINTS

GIOVANNI FUSCO

In optimal control theory, *infimum gap* means that there is a gap between the infimum values of a given minimum problem and an auxiliary problem, obtained by first extending the set of original controls and then convexifying the extended velocities set. In this talk we present sufficient conditions for the absence of an infimum gap for a wide class of optimal control problems subject to endpoint and state constraints. These conditions, which also guarantee controllability of the original system to an extended solution, are based on a nondegenerate version of the nonsmooth constrained Maximum Principle, expressed in terms of subdifferentials. In particular, under some new constraint qualification conditions based on [1, 2] we prove the following theorem.

Theorem. *If a (relaxed) extended minimizer is a* nondegenerate normal *extremal, i.e. every set of nondegenerate multipliers has cost-multiplier greater than zero, then there is no infimum gap.*

This result enhances the outcomes of the previous literature. In fact, in order to prove that there is no infimum gap, it is sufficient to prove that normality holds not for all sets of multipliers (see for instance [5, 6]), but only among the nondegenerate ones. As a consequence, we are able to include the common case of fixed initial state with active state constraint, situation in which every extremal is abnormal. A final example will capture this circumstance, so that to illustrate our theorem and the improvements with respect to the previous literature mentioned above. See also [3, 4].

This is a joint work with Monica Motta.

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META-HEURISTIC ALGORITHMS FOR A MULTI-ATTRIBUTE VEHICLE ROUTING PROBLEM IN EXPRESS FREIGHT TRANSPORTATION

NICOLA GASTALDON

Freight transportation industry is characterized by several decisional problems that operations managers have to cope with. Not only vehicle routes must be planned before their execution, but also other types of decisions must be taken, in order to answer events that may dynamically occur during operations, as for instance road network congestion or vehicle failures. Trans-Cel, a small trucking company in Padova (Italy), has developed different decision support tools interconnected through a data sharing system. These tools rely on an algorithmic engine that includes a routing optimization algorithm and artificial intelligence systems to support routes operations. The optimization engine solves a Multi-Attribute Vehicle Routing Problem inspired by the Trans-Cel context. Trans-Cel operates a fleet of heterogeneous trucks and offers an express service for requests including multiple pickup and multiple delivery positions spread in a regional area, with associated soft or hard time windows often falling in the same working day. Routes are planned on a daily basis and re-optimized on-the-fly to fit new requests, taking into account constraints and preferences on capacities, hours of service, route termination points. The objective is to maximize the difference between the revenue from satisfied orders and the operational costs. The problem mixes attributes from both intercity less-than-truckload and express couriers operations, and we propose a solution approach based on a two-level local search heuristic. The first level assigns orders to vehicles through a variable neighborhood stochastic tabu search; the second level optimizes the route service sequences. Results have been compared to bounds obtained from a mathematical programming model solved by column generation. Experience on the field and test on literature instances attest to the quality of results and the efficiency of the proposed approach. Computational tests on real instances show that the proposed approach: provides, on average, feasible solutions within 1% from optimal bounds certified by the mathematical model; runs in less than one minute; allows an improvement between 2.2% and 15.3% (9.3% on average) with respect to the operations managers' current practice.

This is a joint work with Prof. Luigi De Giovanni *(Dipartimento di Matematica "Tullio Levi-Civita", Università di Padova) and* Filippo Sottovia *(Il Mio Responsabile Tecnico).*

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HIGHER DIFFERENTIABILITY RESULTS FOR SOLUTIONS TO SOME NON-AUTONOMOUS ELLIPTIC OBSTACLE PROBLEMS

ANDREA GENTILE

The aim of this talk is to present some higher differentiability results for solution to non-autonomous obstacle problems of the form

$$\min \left\{ \int_\Omega f\left(x, Dv(x)\right) dx \,:\, v \in \mathcal{K}_\psi(\Omega) \right\},$$

where $\mathcal{K}_\psi(\Omega)$ is the class of admissible functions and the function f satisfies some growth and ellipticity conditions.
Mostly, the energy density satisfies standard p-growth conditions, with $p \geq 2$ or $1 < p < 2$, but also the case of (p, q)-growth conditions is treated, with $q > p \geq 2$.
The second key point is that the map $x \mapsto D_\xi f(x, \xi)$ belongs to a suitable functional space, and the regularity properties of solutions depend on this space, and on the assumptions on the obstacle ψ.
In [1], [3] and [4], the obstacle and the map $x \mapsto D_\xi f(x, \xi)$ belongs to some Sobolev spaces and ψ is also assumed to be a priori bounded.
In [2] both integer and fractional order higher differentiability results are proved, since both the cases of Sobolev and Besov-Lipschitz coefficients are treated.

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ASYMPTOTIC PROFILE FOR A TWO-TERMS TIME FRACTIONAL DIFFUSION PROBLEM

GIOVANNI GIRARDI

It is well known that differential equations with fractional derivatives turned out to be suitable to describe in a very good way various physical phenomena in areas like rheology, biology, engineering, mathematical physics, etc. One of the open problems in this field is finding some easy and effective methods for solving such equations. Such problem becomes even more difficult when multiple fractional in time derivatives are involved in the equation. In this talk we consider the Cauchy-type problem associated to the time fractional partial differential equation:

$$\begin{cases} \partial_t u + \partial_t^\beta u - \Delta u = g(t,x), & t > 0,\ x \in \mathbb{R}^n, \\ u(0,x) = u_0(x), \end{cases}$$

where the fractional derivative ∂_t^β is in Caputo sense. We provide a sufficient condition on the right-hand term $g(t,x)$ to obtain a solution in $\mathcal{C}_b([0,\infty),H^s)$. We exploit a dissipative-smoothing effect which allows to describe the asymptotic profile of the solution in low space dimension. As a corollary of this latter result, we investigate a class of nonlinear perturbations of the problem, for which global-in-time small data solutions exist and we show that their asymptotic profile is independent on the nonlinear perturbation.

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OPTIMAL GRADIENT REGULARITY FOR SEMILINEAR AND QUASILINEAR EQUATIONS WITH POWER-GROWTH NONLINEARITIES

ALESSANDRO GOFFI

In this talk I will survey on recent developments concerning the optimal gradient regularity properties of solutions to some nonlinear elliptic and parabolic equations with first-order terms having power-like growth and unbounded right-hand side in Lebesgue scales. These results are obtained through integral Bernstein methods and/or duality arguments and answer a conjecture raised by P.-L. Lions on stationary problems [9, 10]. In particular, such approaches allow to encompass viscous Hamilton-Jacobi equations, both in the stationary and parabolic case, see [3, 5] and [1, 2] respectively, problems with nonlocal diffusion [6] as well as equations driven by p-Laplacian operators [4]. Finally, I will discuss the implications of these results to the regularity theory of Mean Field Games systems introduced by J.-M. Lasry and P.-L. Lions [8], see [2, 7].

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RANDOM WAVE MODELS ON HYPERBOLIC SPACE

FRANCESCO GROTTO

Berry's model is a Gaussian random field on Euclidean space describing the high-frequency behaviour of Laplacian eigenfunctions and it was originally introduced in the study of chaotic quantum billiards on flat domains [2, 1, 5]. Gaussian random waves, that is Gaussian random fields whose samples are Laplacian eigenfunctions, have also been the object of extensive research in other geometrical settings, such as on the sphere S^2 and on tori (arithmetic random waves). More generally, results concerning local behaviour of Gaussian random fields on compact manifolds have also been obtained. (Refer to the recent [3, 4, 5] for a broader overview).

After a brief outline of previous results, we will describe how it is possible to define an analogous Gaussian model on hyperbolic plane (and also in higher dimension), moving a step forward towards non-compact negatively curved surfaces, a classical geometric setting for chaotic dynamics. We will outline how high-frequency and large-domain limiting behaviours of random waves in this new setting mirror the ones on Euclidean space, notwithstanding the different underlying geometry. We will in particular discuss nonlinear functionals such as excursion areas and the length of nodal sets, their moments, their Wiener chaos expansions and CLTs they satisfy.

This is a joint work with Giovanni Peccati (Université du Luxembourg).

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STRONGLY SINGULAR CONVECTIVE ELLIPTIC EQUATIONS IN $\mathbb{R}^N$ DRIVEN BY A NON-HOMOGENEOUS OPERATOR

UMBERTO GUARNOTTA

The aim of this talk is to present an existence result for the following problem:

$$\text{(P)} \qquad \begin{cases} -\operatorname{div} a(\nabla u) = f(x,u) + g(x,\nabla u) & \text{in } \mathbb{R}^N, \\ u > 0 & \text{in } \mathbb{R}^N, \end{cases}$$

where $N \geq 2$. The differential operator $u \mapsto \operatorname{div} a(\nabla u)$, usually called a-Laplacian, is patterned after the (p,q)-Laplacian $\Delta_p + \Delta_q$, $1 < q < p < +\infty$. The reaction terms $f : \mathbb{R}^N \times (0,+\infty) \to [0,+\infty)$ and $g : \mathbb{R}^N \times \mathbb{R}^N \to [0,+\infty)$ are Carathéodory functions obeying suitable growth conditions.

Problem (P) possesses at least four interesting peculiarities:

- the operator $u \mapsto a(\nabla u)$ can be non-homogeneous;
- the reaction term f is strongly singular (i.e., it behaves like $u^{-\gamma}$ with $\gamma \geq 1$) and $f(x,\cdot)$ can be non-monotone;
- the reaction term g is convective (i.e., it depends on ∇u);
- the problem is set in the whole space $\mathbb{R}^N$.

Problems exhibiting some of these features arise from applications, in particular in Chemistry and Biology, and have been recently investigated by many authors from different viewpoints, as existence, regularity, and qualitative properties of solutions. To the best of our knowledge, the result presented here represents the first contribution about existence of solutions to quasi-linear singular convective problems in the whole space.

From a technical point of view, several challenges arise in this framework: (i) non-homogeneity of the differential operator prevents to exploit standard procedures in the construction of sub-solutions, and regularity issues do not allow to work directly in the whole space; (ii) due to the strongly singular nature of the problem, gaining compactness from energy estimates requires some efforts, such as localization procedures and fine energy estimates on suitable super-level sets of solutions; (iii) the loss of variational structure compels to use topological and monotonicity methods instead of variational ones; (iv) the setting $\mathbb{R}^N$ causes lack of compactness for Sobolev embeddings.

A variety of techniques will be used to ensure the existence of a generalized solution u (that is, a distributional solution such that $\operatorname{essinf}_K u > 0$ for any compact $K \subseteq \mathbb{R}^N$): in addition to the aforementioned tools, also regularization and approximation procedures (such as the shifting method), sub-super-solution technique, fixed point and regularity theory, maximum and comparison principles, as well as some *ad hoc* compactness results, are employed.

This is a joint work with Laura Gambera.

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THE ASYMPTOTIC LAKE EQUATIONS FOR VANISHING OR EMERGING ISLANDS

LARS ERIC HIENTZSCH

The lake equations arise as a 2D shallow water model describing the evolution of the vertically-averaged horizontal component of the 3D velocity field. A lake is characterized by the geometry of its surface and its depth function. The equations are degenerate if vanishing topographies are included in the description.

Motivated by physical phenomena such as flooding, sedimentation and seismic activity, we investigate the stability of these equations under changes of both the geometry and the topography.

More precisely, we consider different scenarios leading to the appearance of a degenerate island, consisting of one point and zero depth, for which the lake equations become singular. In the singular limit of an island collapsing to a point we prove that the asymptotic lake equations include an additional point vortex. Second, we discuss the scenario of an emergent island.

We highlight differences and analogies to the small obstacle problem for the 2D incompressible Euler equations (lake equations with flat topography). New uniform estimates in weighted spaces are introduced that enable us to prove the compactness result.

This is joint work with Christophe Lacave and Evelyne Miot (Université Grenoble Alpes).

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SCALABLE AND PARALLEL NON-LINEAR SOLVERS FOR THE CARDIAC BIDOMAIN SYSTEM

NGOC MAI MONICA HUYNH

The development of effective and scalable solvers for the solution of mathematical models of the cardiac electro-mechanical activity has increasingly grown in the last decade. In particular, modern computational architectures have enlarged the possibility to run large-scale simulations within reasonable computational times. In any case, the multiscale systems arising from discretizations of such models has required the development of specific techniques which can balance accuracy in the solution while being computationally competitive (in terms of efficiency and scalability).

In this talk, we focus on the numerical simulation of the cardiac electrical activity, by solving the Bidomain equations, a system of nonlinear parabolic reaction-diffusion equations describing the propagation of the electric impulse in the cardiac tissue, coupled with a ionic membrane model. The numerical solution of the resulting model represents a challenging task, due to the coupling of macroscopic and microscopic phenomena (the propagation of the electric signal in the cardiac tissue and the ionic currents dynamics at cellular level, respectively).

By means of a staggered approach, where we decoupled the two phenomena, and by applying a finite element discretization in space and the Backward Euler scheme in time, we face the solution of a non-linear algebraic system. We propose here for its solution several parallel non-linear solvers such as non-linear Conjugate Gradient or quasi-Newton methods.

We analyze the theoretical convergence of the considered methods and extensive parallel numerical tests show scalability and efficiency of the proposed approach. We investigate robustness considering both phenomenological and human ventricular ionic models.
These results provide a basis for further studies of parallel solvers for cardiac electrophysiology models that combine parallel efficiency while yielding accuracy in the solution.

This is a joint work with N.A.Barnafi (Univ. Pavia), L.F.Pavarino (Univ.Pavia) and S.Scacchi (Univ.Milano).

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ON NON COEXISTENCE OF 2 & 3–RATIONAL CAUSTICS IN NEARLY CIRCULAR BILLIARD TABLES

EDMOND KOUDJINAN

A famous Birkhoff conjecture states that the only integrable convex planar billiards are billiards in an ellipse. We examined two closely related rigidity questions. A rational caustic is a caustic associated to a family of periodic orbits of the same period and the same rotation number. For example, a convex domain with a rational caustic of period two is a domain of a constant width. We investigated a question proposed by Tabachnikov: are there nearly circular domains other than discs with two rational caustics of a prime period p and q? In this talk, I will discuss our following two new results:

- (rigidity) There are no nearly circular domains with two coexisting rational caustics of period two and three.
- (no super-rigidity) There may be infinitely many deformations of the circular domains with two coexisting rational caustics of period three and five with error given by the 3rd power of the perturbation parameter.

This is based on a joint work with Vadim Kaloshin.

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EQUIVARIANT LOCALIZATION METHODS, ORIENTATIONS AND MODULARITY

EUGENIO LANDI

Atiyah, in [1], using ζ-regularization techniques and the equivariant localization theorem, recovered the $\hat{A}$-genus of a Spin manifold as a class in equivariant cohomology. In particular he showed that (roughly)

$$\hat{A}(M) = \int_M \frac{1}{\mathrm{eul}(\nu)}$$

where ν is the (infinite rank, S^1-equivariant) normal bundle of $\iota : M \to \mathcal{L}M = \mathrm{Maps}(S^1, M)$ and $\mathrm{eul}(\nu)$ is its Euler class.

In [2], we generalize this fact to the space of maps from elliptic curves and to the Witten genus. In particular, introducing a suitable "anti-holomorphic sector" we prove a generalization of the equivariant localization theorem. This allows us to consider smooth actions of elliptic curves $\mathbb{C}/\Lambda$ and thus lets us apply ζ-regularization techniques, with rigour not yet found in the literature, to recover the Witten genus as the modular form given (again, roughly), over a point $\tau \in \mathbb{H}$, by the formula

$$\mathrm{Wit}(M)(\tau) = \int_M \frac{1}{\mathrm{eul}(\nu_\tau)}$$

where ν_τ is the (infinite rank, $\mathbb{C}/\Lambda$-equivariant) normal bundle of $\iota : M \to \mathrm{Maps}(\mathbb{C}/\Lambda, M)$, with Λ the lattice generated by 1 and τ, M a rational String manifold, and $\mathrm{eul}(\nu)$ the Euler class of ν.

After a brief introduction of the localization theorem in both the classical and antiholomorphic setting, I will survey some ζ-regularization theory and give precise statements of both Atiyah's result and ours.

This is a joint work with Mattia Coloma and Domenico Fiorenza.

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NONLOCAL MINIMAL GRAPHS

LUCA LOMBARDINI

The fractional perimeter and the associated minimizing sets, whose boundaries are the so-called nonlocal minimal surfaces, were introduced by Caffarelli, Roquejoffre and Savin in [1], in 2010, and have since then attracted a lot of interest.

In this talk I will present some recent results, that I have obtained with Matteo Cozzi, in [2], concerning subgraphs that have finite fractional perimeter. A preliminary version of [2] can be found in my PhD thesis [3].

We will define a fractional and nonlocal counterpart of the classical area functional, then we will focus on studying its minimizers, the nonlocal minimal graphs. In particular, I will prove the existence and uniqueness of minimizers, under very mild assumptions on the prescribed exterior datum. By exploiting a vertical rearrangement inequality, I will then show that the subgraphs of the minimizers of the fractional area functional minimize the fractional perimeter. Finally, I will give a sketch of the proof of the equivalence of nonlocal minimal graphs and solutions of the fractional mean curvature equation.

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A LOCAL EMBEDDED METHOD FOR FLOW IN FRACTURED POROUS MEDIA WITH NUMERICAL UPSCALING AND MACHINE LEARNING

DAVIDE LOSAPIO

The study of flow in fractured porous media is a key ingredient for many geoscience applications, such as reservoir management. Modelling and simulation of these highly heterogeneous and geometrically complex systems require the adoption of non-standard numerical schemes.

The Embedded Discrete Fracture Model (EDFM) [1] is a simple and effective way to account for fractures with coarse and regular grids, but it suffers from some limitations: the expression for the flux interaction terms between porous matrix and fractures comes from the assumption of linear pressure distribution around fractures, which holds true only far from the tips and fracture intersections, and it can be employed for highly permeable fractures only.

We propose an improvement of EDFM, i.e. the Local Embedded Discrete Fracture Model (LEDFM), which aims at overcoming both its limitations computing an improved coupling between fractures and the surrounding porous medium by a) relaxing the linear pressure distribution assumption, b) accounting for impermeable fractures modifying near-fracture transmissibilities.

These results are achieved by adopting local flow-based upscaling methods to compute new transmissibilities for matrix-fracture and near-fracture matrix-matrix connections. Here the coarse model coincides with an embedded model, whereas in the numerical upscaling techniques for fracture networks found in the literature the coarse model typically belongs to the family of Continuum Fracture Models (CFM), that are not capable of explicitly representing fractures. The definitions of the local fine scale problems for transmissibility computation are inspired from the aforementioned techniques, and a conforming method is used to solve them.

In some cases, a higher accuracy for the description of the near fracture flow is needed, so that the local problems for the computation of matrix-matrix transmissibilities are replaced with a multiscale approach.

Generally, a high number of local problems should be solved. Hence, to speed up an otherwise very costly procedure, neural networks are integrated in the model to provide a fast evaluation of the local flow problems. Indeed, these are solved in an offline stage, where different fracture configurations and matrix-fracture permeability contrasts are examined. The results obtained are then used to train two feedforward neural networks, whose goal is that of learning the transmissibility functions relative to the two different local problems. The transmissibilities are then obtained by evaluating the networks instead of solving the local problems directly, resulting in a dramatic reduction of the computational cost of the method.

The results obtained from several numerical tests, comparing the solutions of different embedded methods, including the newly developed LEDFM, with the reference ones, show that the local method overcomes both the limitations mentioned before pertaining to the classic EDFM method.

This is a joint work with Anna Scotti.

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DIRECTION-DEPENDENT TURNING LEADS TO ANISOTROPIC DIFFUSION AND PERSISTENCE

NADIA LOY

Cells and organisms follow aligned structures in their environment, a process that can generate persistent migration paths. In particular, movement of cells through tissues is critical during both healthy and pathological processes. Embryonic development relies on cells migrating from origin to final tissue destination, repair processes necessitate movement of fibroblasts and macrophages into the wound site, and migration of cancerous cells, unhappily, leads to tumour invasion and metastasis dissemination. Consequently, there is clear reason to understand the factors that guide cells with one such process, contact guidance, defining the movement of cells along linear/aligned tissue features, for example, blood vessels, white matter brain fibres, or the collagen fibres of connective tissue.

Kinetic transport equations are a popular modelling tool for describing biological movements at the mesoscopic level, yet their formulations usually assume a constant turning rate. Here we relax this simplification, extending to include a turning rate that varies according to the anisotropy of a heterogeneous environment. In particular we extend the very well known transport model for contact guidance proposed by Hillen [2] and we extend known methods of parabolic and hyperbolic scaling. We apply the results to cell movement on micro-patterned domains also through numerical simulation of the transport model. We show that inclusion of orientation dependence in the turning rate can lead to persistence of motion in an otherwise fully symmetric environment, and generate enhanced diffusion in structured domains.

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HEAT EQUATION AND LAYER POTENTIALS: OLD AND NEW RESULTS

PAOLO LUZZINI

Layer potentials are a powerful and versatile tool to analyze boundary value problems. They provide an integral representation of the solution and allow transforming the problem to an integral equation defined on the boundary so, in this way, one can take advantage of the theory of integral equations and operators. Also, casting the problem on the boundary reduces the dimension by one making the numerical computations faster.

While the elliptic framework is better understood, its parabolic counterpart remains somehow less investigated. In this talk I will first introduce the layer potentials associated with the heat operator and I will review some classical contributions. Then I will present some new results concerning the regularity of the integral operators associated with layer heat potentials and finally I will show a few application to boundary value problems.

Part of the results presented in the talk have been obtained in collaboration with Prof. Lanza de Cristoforis (Università degli Studi di Padova) and Prof. Dalla Riva (Università degli Studi di Palermo)

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ADAPTIVE HIGH ORDER WELL BALANCED COMPACT APPROXIMATE METHOD FOR SYSTEMS OF CONSERVATION AND BALANCE LAW

EMANUELE MACCA, CARLOS PARES, AND GIOVANNI RUSSO

Compact Approximate Taylor (CAT) methods for systems of conservation laws were introduced by Carrillo and Parés in 2019. These methods, based on a strategy that allows one to extend high-order Lax-Wendroff methods to nonlinear systems without using the Cauchy-Kovalevskaya procedure, have arbitrary even order of accuracy $2p$ and use $(2p+1)$-point stencils, where p is an arbitrary positive integer. More recently in 2021 Carrillo, Macca, Parés, Russo and Zorío introduced a strategy to get rid of the spurious oscillations close to discontinuities produced by CAT methods. This strategy led to the so-called Adaptive CAT (ACAT) methods, in which the order of accuracy – and thus the width of the stencils – is adapted to the local smoothness of the solution. In this talk we discuss about the extension of CAT and ACAT methods to systems of balance laws. To do this, the source term is written as the derivative of its indefinite integral that is formally treated as a flux function. The well-balanced property of the methods is discussed and a variant that allows in principle to preserve any analytically stationary solution is presented.

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GENERALIZED HEEGNER CYCLES AND p-ADIC L-FUNCTIONS IN A QUATERNIONIC SETTING

PAOLA MAGRONE

A central topic in algebraic number theory is the theory of Galois representations, continuous group homomorphisms from a Galois group to the group of automorphisms of a vector space. To a suitable Galois representation one can attach two fundamental objects: an L-function and a Selmer group. *L-functions* are particular complex functions which have a special role in number theory. They arise in various situations: one can attach an L-function also to a number field, an elliptic curve or more in general an abelian variety, a modular form, a Dirichlet or a Hecke character. A typical example is the famous Riemann zeta function, which is known to contain information about the prime numbers. More in general, L-functions are expected to encode a lot of information on the arithmetic objects they are attached to. The *Selmer group* is instead an algebraic object, which is an invariant of the representation that contains deep number theoretical information about it. Even though these two objects have really different natures, they are related by the *Bloch-Kato Conjecture* which, in the cases of our interest, predicts an equality between the order of vanishing of the L-function at a certain point and the dimension of the Selmer group.

In [3], Castella and Hsieh proved significant results about Selmer groups associated with Galois representations attached to newforms (suitables modular forms) twisted by Hecke characters of an imaginary quadratic field. These results lead also to prove other instances of the rank 0 case of the Bloch–Kato Conjecture for L-functions of modular forms. The key point of the work of Castella and Hsieh is a remarkable link between certain arithmetic objects called *generalized Heegner cycles* that were introduced by Bertolini, Darmon and Prasanna in [1] and suitably defined p-adic L-functions, which are instead objects of p-adic analytic nature, interpolating special values of complex L-series.

All these results are obtained under the so-called *Heegner hypothesis* that the imaginary quadratic field must satisfy with respect to the level of the modular form. What happens if one weakens the Heegner hypothesis, considering more quadratic fields? In this talk, we answer to that question and see that several of the results of Castella–Hsieh can be extended to a quaternionic setting, which is the setting that arises when one works under a "relaxed" Heegner hypothesis. This can be done working with *generalized Heegner cycles* over Shimura curves (instead of modular curves) introduced by Brooks in [2]. For all the details, see [4].

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VARIATIONAL CONVERGENCES FOR FUNCTIONALS AND DIFFERENTIAL OPERATORS DEPENDING ON VECTOR FIELDS

ALBERTO MAIONE

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain. The X-gradient is a family of Lipschitz continuous vector fields $X = (X_1, .., X_m)$ $(m \leq n)$ that are pointwise linearly independent, outside Lebesgue measure zero sets. The Sobolev spaces associated with the X-gradient are

$$W_X^{1,p}(\Omega) := \{u \in L^p(\Omega) : X_j u \in L^p(\Omega) \text{ for } j = 1, \ldots, m\}$$

and $W_{X,0}^{1,p}(\Omega)$, defined as the closure of $\mathbf{C}_c^1(\Omega) \cap W_X^{1,p}(\Omega)$ in $W_X^{1,p}(\Omega)$. We are interested in families X satisfying a global Poincaré inequality and such that $W_{X,0}^{1,p}(\Omega)$ compactly embeds into $L^p(\Omega)$, $p \in [1, \infty)$. For such families, the following Γ-compactness result will be showed.

Theorem 1. *Let $1 < p < \infty$ and define the sequence $F_h : L^p(\Omega) \to \mathbb{R} \cup \{\infty\}$, $h \in \mathbb{N}$, by*

$$F_h(u) := \begin{cases} \int_\Omega f_h(x, Xu(x))dx & \text{if } u \in W_X^{1,p}(\Omega) \\ +\infty & \text{if } u \in L^p(\Omega) \setminus W_X^{1,p}(\Omega) \end{cases},$$

where $f_h : \Omega \times \mathbb{R}^m \to \mathbb{R}$ is a Carathéodory function, convex w.r.t. the second variable, satisfying

$$c_0|\eta|^p - a_0(x) \leq f(x, \eta) \leq c_1|\eta|^p + a_1(x) \quad \text{for a.e. } x \in \Omega \text{ for any } \eta \in \mathbb{R}^m,$$

with $a_0, a_1 \in L^1(\Omega)$ nonnegative and $c_0 \leq c_1$ positive constants, and Borel-measurable on Ω.

Then, there exist $f : \Omega \times \mathbb{R}^m \to \mathbb{R}$, satisfying the same hypotheses of f_h (with the same constants and L^1 functions) and $F : L^p(\Omega) \to \mathbb{R} \cup \{\infty\}$ such that (up to subsequences)

1) *F_h Γ-converges to F in the strong topology of $L^p(\Omega)$, as $h \to \infty$;*
2) *the limit F can be represented by*

$$F(u) := \begin{cases} \int_\Omega f(x, Xu(x))dx & \text{if } u \in W_X^{1,p}(\Omega) \\ +\infty & \text{if } u \in L^p(\Omega) \setminus W_X^{1,p}(\Omega) \end{cases}.$$

As a consequence of the previous result, we show that the class of linear differential operators in X-divergence form is closed in the topology of the H-convergence, by adapting a variational approach introduced by Ansini, Dal Maso and Zeppieri [1].
This is a joint work with Andrea Pinamonti, Francesco Serra Cassano (University of Trento) [4, 5], Fabio Paronetto (University of Padova) and Eugenio Vecchi (University of Bologna) [3].

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LINEAR DYNAMICS: AN ANALYSIS OF THE BEHAVIOR OF COMPOSITION OPERATORS ON L^p SPACES

MARTINA MAIURIELLO

Linear Dynamics is an area of mathematics, lying in the intersection of Operator Theory and Dynamical Systems and consisting in the study of the behavior of the iterates of linear operators. It has received a lot of attention in the last decades and a flurry of intriguing results have been produced.
A meaningful, under the dynamical point of view, class of operators, is that of Composition Operators, $T_f : \varphi \to \varphi \circ f$, where it is the nature of the transformation f to determine the dynamics.

In the talk, in the setting of separable Banach spaces, I will briefly recall some fundamentals of Linear Dynamics, like hypercyclicity, mixing, Devaney and Li-Yorke chaos, frequent hypercyclicity, generalized hyperbolicity, expansivity and shadowing. These properties are completely characterized for a significant class of operators, the *weighted shifts*, and they have also been recently investigated for composition operators on L^p spaces, because of their versatility in applications to several research areas.
In the first part of the talk, after an excursion on the state of the art of these topics, I will focus on a large natural class of composition operators: I will prove that the notions of generalized hyperbolicity and shadowing coincide for such class, and I will also describe computational tools which allow the construction of operators with and without the shadowing property.
In the second part of the talk, I will show a general technique which allows to lift up the characterizations given for weighted backward shifts to a broader class of operators on L^p spaces, the *shift-like operators*, which naturally appear as composition operators on L^p when the underlying space is dissipative.

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SEMIOSPHERE LENS FOR MATHEMATICS TEACHER EDUCATION

CAROLA MANOLINO

In Italy, there are no compulsory contents or practices for mathematics teachers' professional development. The results of the 2018 OECD Teaching and Learning International Survey, which aims at examining the level of professionalism in teaching, show that teachers' education have a positive impact, a change, on teaching practice. However, "prescription does not motivate all teachers to change their practices, [...] providing only a body of knowledge might leave some teachers not knowing what direction they should take to enact changes" ([1], p.2). Furthermore,

1. only 44% of teachers affirm to participate in training based on networking, even if they also report that professional development based on collaboration and collaborative approaches to teaching is among the most impactful for them;
2. around half of teachers and principals report that their participation in the professional development available to them is restricted by scheduling conflicts and lack of incentives;
3. changes in teaching and learning practices "are a challenge", due to the fact that we are dealing with cultural, psychological, pedagogical ([3], p. 230) and disciplinary issues.

Universities and researchers are responsible for addressing such issues and proposing appropriate measures. Scholars provide the epistemological legitimacy of the taught knowledge. Indeed, any human activity "has its own scholars, held 'to know best' than the rest of the people in the little world where they belong" ([4], p. 76). This is particularly pronounced in the Italian academic tradition where general pedagogy aspects are traditionally separated from problems of teaching science and mathematics. Knowing mathematics is different from knowing how to teach mathematics. There is a need for sustainable mathematics teacher education, but also for research into cultural and social aspects to enable mathematics teachers' critical reflection.

I address the issue using Lotman[5]'s Semiosphere lens. My study is based on a semiotic-cultural analysis of data, collected during Lesson Study experiences in mathematics [2], i.e. a Chinese teacher professional development methodology that can trigger a critical reflection. If observed through the theoretical lens of Cultural Transposition [6], Lesson Study as a foreign methodology questions teaching intentionalities of Italian mathematics teachers. The analysis of two experiences carried out respectively with grade 5 and grade 12 students, in the area of calculus of probability, are paradigmatic for this study and demonstrate the essential activation of reflective practices around the mathematical concept itself as well as teaching practice.

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CONTOUR INTEGRAL METHODS AND CERTIFIED REDUCED BASIS FOR PARAMETRIC DYNAMICAL PROBLEMS

MATTIA MANUCCI

We discuss a reduced bases method [3] for linear evolution PDEs, which is based on the application of the Laplace transform [4]. The main advantage of this approach consists in the fact that, differently from time stepping methods, like Runge-Kutta integrators, the Laplace transform allows to compute the solution directly at a given instant, which can be done by approximating the contour integral associated to the inverse Laplace transform by a suitable quadrature formula [1, 2]. In terms of the reduced basis methodology, this determines a significant improvement in the reduction phase, like the one based on the classical proper orthogonal decomposition (POD), since the number of vectors to which the decomposition applies is drastically reduced as it does not contain all intermediate solutions generated along an integration grid by a time stepping method. We show by some illustrative parabolic PDEs arising from finance the effectiveness of the method and also provide some evidence that the method we propose, when applied to a simple advection equation, does not suffer the problem of slow decay of singular values which instead affects methods based on time integration of the Cauchy problem.

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VARIABLY SCALED (DISCONTINUOUS) KERNELS: FROM IMAGE RECONSTRUCTION TO SUPERVISED LEARNING

FRANCESCO MARCHETTI

Kernel methods are widely-employed and successfully applied in many fields. In this presentation, we discuss the employment of Variably Scaled Discontinuous Kernels (VSDKs) in the approximation of discontinuous functions and of Variably Scaled Kernels (VSKs) in the context of supervised learning. Variably scaled kernels have been introduced in the context of approximation theory in 2015. Here, we introduce VSDKs and we prove their efficacy in the reduction of the so-called Gibbs phenomenon, which affects the reconstruction of discontinuous functions causing non-physical oscillations and artifacts [3]. Moreover, we discuss the usage of VSDKs in medical imaging, more precisely in the context of magnetic particle imaging [2]. Then, we present an extension of the variably scaled setting to the support vector machines framework. In this case, we show how the resulting stacking technique is capable of improving the classification accuracy of standard methods [1]. The proposed technique is then applied in concrete medical classification tasks [4].

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KOLMOGOROV EQUATIONS ON SPACES OF MEASURES ASSOCIATED TO NONLINEAR FILTERING PROCESSES

MATTIA MARTINI

Backward Kolmogorov equations are partial differential equations of parabolic type with given final condition. The relation among them and certain stochastic processes has been intensively investigated in both finite and infinite dimensional case.

The aim of this talk is to introduce a class of backward Kolmogorov equations on spaces of probability and positive measures, associated to measure-valued stochastic processes arising in the context of nonlinear filtering. Indeed, in the filtering framework one can formulate two stochastic differential equations, called Zakai and Kushner-Stratonovich equations, that are satisfied by a positive measure and a probability measure-valued process respectively. Thus, one can study the associated backward Kolmogorov equations, that are partial differential equations of parabolic type on the space of measures.

In the literature, the Kolmogorov equations associated to nonlinear filtering processes have been studied assuming that the measure-valued processes admit a density and then by exploiting stochastic calculus techniques in Hilbert spaces. The approach used here differs from that one, since the existence of a density is not assumed and everything is done directly in the context of measures.

In the talk, we will introduce tools that allow us to write the backward Kolmogorov equations on spaces of measures and then present an existence and uniqueness result for classical solutions. If it remains time, we will discuss also a well posedness result for viscosity solutions.

The talk in based on the preprints [1, 2].

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A TIME-DEPENDENT SWITCHING MEAN-FIELD GAME ON NETWORKS

LUCIANO MARZUFERO

Motivated by an optimal visiting problem, we investigate a switching mean-field game model on a network, where both a decisional and a switching time-variable are controls at disposal of the agents for what concerns, respectively, the instant to decide and to perform the switch. Every switch between the nodes of the network corresponds to a flip from 0 to 1 of one component of the string $p = (p_1, \dots, p_n)$ which, in the optimal visiting view, possibly represents the visited targets, being labeled by $i = 1, \dots, n$. The goal is to reach the final string $(1, \dots, 1)$ (i.e., to visit all the targets) within a fixed final time T, minimizing a switching cost also depending on the congestion on the nodes. We show the existence of a suitable approximated ε-mean-field equilibrium and then address the limit when ε goes to 0.

This is a joint work with Fabio Bagagiolo. The main reference is preprint [1], submitted in December 2021. In the *References* section, you can find all the bibliography concerning the topic.

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ZONOIDS: WHAT ARE THEY AND HOW TO MULTIPLY THEM

LÉO MATHIS

Zonoids are a particular class of convex bodies, they are defined as limits of finite Minkowski sum of line segments. I will explain the different terms of this definition (limit, Minkowski sum...). I will show that zonoids are easily constructed using random vectors following a method introduced by Vitale. I will then present what I like to call the Fundamental Theorem of Zonoid Calculus, joint work with P.Burgisser, P.Brieding & A. Lerario [1], which allows to build multilinear operations on the space of zonoids. In particular this allows to define a product on the space of zonoids of the exterior algebra and hence define the *zonoid algebra*, see Figure 1. I will show how this algebra computes random intersections in stochastic geometry, this is a work in progress with P.Burgisser, P.Brieding & A. Lerario and with M. Stecconi.

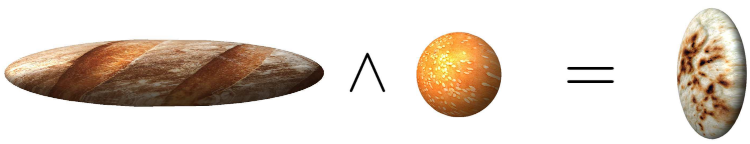

FIGURE 1. The wedge product of zonoids illustrated with bread

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SISSA

RECENT DEVELOPMENTS OF THE SET-THEORETICAL SOLUTIONS TO THE PENTAGON EQUATION

MARZIA MAZZOTTA

The pentagon equation plays an important role in the modern Mathematical Physics, especially in the area of quantum field theory, and it is widely investigated in various contexts. A brief introduction to this topic is contained, for instance, in [3].
Given a set X, a *set-theoretical solution of the pentagon equation*, or briefly a *PE solution*, is a map $s : X \times X \to X \times X$ satisfying the relation

$$s_{23}\, s_{13}\, s_{12} = s_{12}\, s_{23},$$

where $s_{12} = s \times id_X$, $s_{23} = id_X \times s$, and $s_{13} = (id_X \times \tau)s_{12}(id_X \times \tau)$, with $\tau(x,y) = (y,x)$. First examples of PE solutions can be extrapolated in the pioneering work of Kashaev and Sergeev [5], where it is proved that the unique bijective solution on a group G is given by $s(x,y) = (xy, y)$. Recently, our attention has been posed on the study of of this equation, from a purely algebraic point of view [1].

In this talk, we present the complete description of not bijective PE solutions of the form $s\,(x,y) = (xy,\, \theta_x(y))$ on a group G, where θ_x are maps from G into itself, given in [1, Theorem 15]. Such a description involves normal subgroups of the group G, since the set $K = \{x \in G \mid \theta_1(x) = 1\}$ is a normal subgroup of G, despite, in general, θ_1 is not a homomorphism of G.
Moreover, as developed in [2], we show how PE solutions are useful to find set-theoretical solutions of the Yang-Baxter equation, another basic equation of Mathematical Physics, for which in the '90s Drinfel'd [4] posed the problem of finding all its solutions. In particular, we provide a construction of YBE solutions involving PE solutions and specific classes of semigroups by showing that these maps are different from those known until now.

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SMOOTHING EFFECTS AND INFINITE TIME BLOWUP FOR REACTION-DIFFUSION EQUATIONS: AN APPROACH VIA SOBOLEV AND POINCARÉ INEQUALITIES

GIULIA MEGLIOLI

We consider the following Cauchy problem for a class of reaction-diffusion equations

$$\begin{cases} u_t = \Delta u^m + u^p & \text{in } M \times (0,T) \\ u = u_0 & \text{in } M \times \{0\} \end{cases} \tag{1}$$

where M be a complete noncompact Riemannian manifold of infinite volume. Here $m > 1$, which is known as slow diffusion of the porous medium type. We consider the particularly delicate case $1 < p < m$ in problem (1), a case largely left open in [1] even when the initial datum is smooth and compactly supported. We prove global existence for L^m data, and a smoothing effect for the evolution, i.e. that solutions corresponding to such data are bounded at all positive times with a quantitative bound on their L^∞ norm. As a consequence of this fact and of a result of [1], it follows that on Cartan-Hadamard manifolds with curvature pinched between two strictly negative constants, solutions corresponding to sufficiently large L^m data give rise to solutions that blow up pointwise everywhere in infinite time, a fact that has no Euclidean analogue. The methods of proof of the smoothing effect are functional analytic in character, as they depend solely on the validity of the Sobolev inequality and on the fact that the L^2 spectrum of Δ on M is bounded away from zero (namely on the validity of a Poincaré inequality on M).

This is a joint work with Gabriele Grillo and Fabio Punzo.

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THREE-DIMENSIONAL PHYSICS-BASED NUMERICAL SIMULATIONS OF EARTHQUAKE GROUND MOTION FOR ADVANCED SEISMIC RISK ASSESSMENT

LAURA MELAS

Three-dimensional physics-based simulations (PBS) represent one of the most powerful techniques for the prediction of seismic wave propagation phenomena. Indeed, based on a high-fidelity level of inputs, they can provide a complete and reliable picture of the seismic wave propagation phenomenon. Here, we are interested to employ PBS to predict seismic response of structures by interfacing ground motion at a site with specific vulnerability model.
In this talk we introduce the mathematical and numerical models for the coupling of the ground motion induced by earthquakes with the induced structural damages of buildings. In order to simulate seismic wave propagation we employ the discontinuous Galerkin spectral element method [1, 2] implemented in the open-source code SPEED (`http://speed.mox.polimi.it`), whereas prediction models of structural damages are based either on empirical laws (fragility curves) or deterministic approaches (linear and non-linear differential models).
The first proposed coupled approach based on fragility curves is then tested considering synthetic physics-based scenarios with earthquake magnitude in the range $6.5 - 7.3M_w$ in the Beijing metropolitan area (China) focusing on the class of high-rise buildings [3].
In the second proposed approach, three-dimensional physics-based scenarios of the 1999 M_w6 Athens earthquake are carried out to study the seismic response of the the Acropolis hill and of the Parthenon. In particular we model the main Greek cultural heritage within the framework of the structural analysis [4].

This research was carried out during my PhD program at Department of Mathematics, Politecnico di Milano, Italy.
This is a joint work with P. F. Antonietti, C. Cauzzi, I. Mazzieri, R. Paolucci, A. Quarteroni, C. Smerzini and M. Stupazzini.

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ASYMPTOTIC BEHAVIOR OF SOLUTIONS TO HAMILTON-JACOBI-BELLMANN EQUATIONS

CRISTIAN MENDICO

The analysis of the ergodic behavior of solutions to Hamilton-Jacobi-Bellmann equations has a long history going back to the seminal paper by [Lions, P.-L., Papanicolaou, G. and Varadhan, S.R.S]. Since this work, the subject has grown very fast and when the Hamiltonian is of Tonelli type a large number of results have been proved. However, few results are available if the Hamiltonian fails to be Tonelli, i.e., the Hamiltonian is neither strictly convex nor coercive with respect to the momentum variable. In particular, such results cover only some specific structure and so, the general problem is still open. In this talk, I will present some recent results obtained in collaboration with Piermarco Cannarsa and Pierre Cardaliaguet concerning the long time-average behavior of solutions to Hamilton-Jacobi-Bellman equations arising from optimal control problems with control of acceleration, first, and then from optimal control problems of sub-Riemannian type. We will finally present some open problems related to the topic.

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REDUCED CONVOLUTIONAL NEURAL NETWORKS FOR IMAGE RECOGNITION AND OBJECT DETECTION

LAURA MENEGHETTI

Computer Vision is a thriving field increasingly exploited in several scientific and engineering contexts in order to solve complex tasks such as the recognition and detection of objects inside pictures. A possible approach to deal with these problems is represented by Convolutional Neural Networks (CNNs), a deep learning algorithm which is able to distinguish the different features of objects thanks to its deep architecture. If on the one hand this deep and complex structure manages to solve difficult tasks, on the other it leads to a highly number of parameters that needs to be calibrated during the training phase. When it comes to practical applications, especially in industries, several issues may arise, since the aforementioned networks may need to operate in embedded systems with limited hardware. The resulting memory constraints have led to the creation of a reduction strategy for the development of a reduced version of an Artificial Neural Network and in particular of a Convolutional Neural Network.
In our work [2] we propose an extension of [1] for dimensionality reduction of CNNs by developing a reduction technique based on methods widely used in the context of Reduced Order Modeling, as Proper Orthogonal Decomposition (POD) and Active Subspaces (AS). The reduced network is obtained by splitting the original one in two different nets connected by the reduction technique: the first one obtained by retaining a certain number of layers of the original CNN and a second one that deals with the classification of the features extracted by the previous part.
We finally provide the numerical results obtained by applying such method to VGG-16 for the problem of image recognition and to SSD-300 for the problem of object detection. In particular we compare the final outcome of the original net with that of its reduced version in terms of final accuracy, memory allocation, speed of the procedure.

This is a joint work with Nicola Demo and Gianluigi Rozza.

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GEOMETRY OF 1-CODIMENSIONAL MEASURES IN THE HEISENBERG GROUPS

ANDREA MERLO

In the Euclidean spaces the notion of rectifiability of a measure is linked to the metric by the celebrated:

Theorem 1 (Preiss, [1]). *Suppose $0 \leq m \leq n$ are integers, ϕ is a Radon measure on $\mathbb{R}^n$ and:*

$$0 < \Theta^m(\phi, x) := \lim_{r \to 0} \frac{\phi(U_r(x))}{r^m} < \infty \qquad \text{at } \phi\text{-almost every } x, \tag{1}$$

where $U_r(x)$ is the Euclidean ball of centre x and radious r. Then ϕ is m-rectifiable, i.e., ϕ-almost all of $\mathbb{R}^n$ can be covered by countably many m-dimensional Lipschtitz submanifolds of $\mathbb{R}^n$.

The most difficult part of the proof of Theorem 1 is to show that the existence of the density, namely that (1) holds, implies that the measure ϕ has flat tangents, i.e.:

$$\mathrm{Tan}(\phi, x) \subseteq \Theta^m(\phi, x)\{\mathcal{H}^m \llcorner V : V \text{ is an } m\text{-plane}\} \qquad \text{at } \phi\text{-almost every point.} \tag{2}$$

The fact that the inclusion (2) implies Theorem 1 is a consequence of the Marstrand-Mattila rectifiability criterion, see for instance [1, Corollary 5.4]. The proof of such inclusion depends on the structure of the Euclidean ball and it is not known whether it is possible to extend it to a general finite dimensional Banach space. The only progress in this direction, to our knowledge, was achieved by A. Lorent, who proved that 2-locally uniform measures in ℓ^3_∞ are rectifiable, see Theorem 5 in [4].

In this talk, I will give present the first extension of Theorem 1 outside Euclidean spaces:

Theorem 2 ([5, 6]). *Suppose ϕ is a Radon measure in $\mathbb{H}^n$ such that:*

$$0 < \Theta^{2n+1}(\phi, x) := \lim_{r \to 0} \frac{\phi(B_r(x))}{r^{2n+1}} < \infty \qquad \text{for } \phi\text{-a.e. } x, \tag{3}$$

where $B_r(x)$ is the Koranyi ball. Then $\mathbb{H}^n$ can be covered ϕ-almost all by countably many $C^1_{\mathbb{H}^n}$-regular surfaces, which are smooth surfaces in a very intrinsic sense and were introduced in [3] *and are fractals from the Euclidean point of view* [2].

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Université de Fribourg

ENGEL CONDITIONS IN GROUPS

MARIALAURA NOCE

The theory of Engel groups plays an important role in group theory, since these groups are closely related to the Burnside problems. In this talk, we survey on Engel elements and Engel groups, and we focus on their development during the last two decades, presenting new results and some open problems.

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SPECTRAL STABILITY OF THE LAPLACIAN UNDER PERTURBATION OF THE BOUNDARY CONDITIONS

ROBERTO OGNIBENE

In this talk I will discuss the behavior of the spectrum of the Laplacian on bounded domains, subject to varying mixed boundary conditions. More precisely, let us assume the boundary of the domain to be split into two parts, on which homogeneous Neumann and Dirichlet boundary conditions are respectively prescribed; let us then assume that, alternately, one of these regions "disappears" and the other one tends to cover the whole boundary. In this framework, I will first describe under which conditions the eigenvalues of the mixed problem converge to the ones of the limit problem (where a single kind of boundary condition is imposed); then, I will sharply quantify the rate of this convergence by providing an explicit first-order asymptotic expansion of the "perturbed" eigenvalues. These results have been obtained in collaboration with Veronica Felli and Benedetta Noris.

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A GREEN'S FUNCTION PROOF OF THE POSITIVE MASS THEOREM

FRANCESCA ORONZIO

In this talk, we describe a new monotonicity formula holding along the level sets of the Green's function of a complete one–ended asymptotically flat manifold of dimension 3 with nonnegative scalar curvature. Using such a formula, I will obtain a simple proof of the following result:

Theorem 1. *Let (M, g) be a 3-dimensional, complete, one–ended asymptotically flat manifold with nonnegative scalar curvature. Then, the ADM mass of (M, g) is nonnegative,*

$$m_{\mathrm{ADM}} \geq 0.$$

Moreover, $m_{\mathrm{ADM}} = 0$ if and only if (M, g) is isometric to $(\mathbb{R}^3, g_{\mathbb{R}^3})$.

See also [1].

This is a joint work with Virginia Agostiniani, Lorenzo Mazzieri.

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MATHEMATICAL MODELS FOR CHROMONIC LIQUID CRYSTALS

SILVIA PAPARINI

This talk is concerned with a peculiar class of lyotropics: chromonic liquid crystals (CLCs). Chromonics in the nematic phase enjoy the head-tail symmetry, that is, they are non-chiral materials with a tendency for their constitutive elements to bundle together so that a director $\boldsymbol{n}$ can be defined at a mesoscopic scale and lacks polarity. The ground state of ordinary nematic liquid crystals is attained when $\boldsymbol{n}$ is uniform in space, while when CLCs are confined in capillary cylinders with degenerate boundary conditions then they are instead observed to acquire a nonuniform arrangement. In particular, their ground state in a cylindrical capillary, often referred to as the *escaped twist* (ET) field is two-fold; it consists of two symmetric twisted configurations (left- and right-handed), each variant occurring with the same likelihood. Despite their peculiar behaviour in 3D, the Frank theory for nematics has been applied to rationalize the experiments with capillary tubes. This is a variational theory which posits a free-energy density quadratic in the director gradient that penalizes all distortions of $\boldsymbol{n}$ away from a uniform alignment. In particular, in a new alternative form, a *double-twist* distortion is identified and corresponds to the elastic constant $(K_{22} - K_{24})$. Ericksen's inequalities ensure that Frank's energy density is positive definite, and the spontaneous emergence of chirality in the nematic texture is not conceivable when they hold. Thus, as expected, Frank's elastic theory justifies the observed configurations of CLCs under cylindrical confinement only if the relevant Ericksen's inequality, $K_{22} \geq K_{24}$, is violated, and so only if Frank's free-energy functional is unbounded below in $3D$ Euclidean space. The negativity of $(K_{22} - K_{24})$ suggests that the pure double-twisted configuration should be the ground state of CLCs in $3D$ space. Unfortunately, this ideal state is only attainable along a $1D$ curve and its extension to a tubular region introduces by necessity a non-uniform texture with pure double twist only along the axis of the cylinder. This is precisely the ET field experimentally observed for CLCs in cylindrical capillaries subject to degenerate boundary conditions. Taking for granted that the ground state of CLCs is an ideal (unattainable), pure double-twist, we may say that the ET field is actually a 'pseudo-ground state'. The following questions then arise: is anything wrong in applying Frank's elastic theory to these materials? Do we need a new elastic theory for CLCs? It follows from a geometric representation for the energy term K_{24} when $\boldsymbol{n}$ is required to obey planar anchoring conditions on the boundary that even when one of the Ericksen's inequalities involving K_{24} is violated, as in this case, the stored Frank's energy is bounded below. Hence, hardly confining boundary conditions could ensure the existence of the minimum also when the free-energy functional is unbounded below. It can thus be legitimate to apply Frank's theory to this particular class of materials under confinement when $K_{22} < K_{24}$. In particular, the local stability of ET field is established through a general formula for the second variation of the free-energy functional. But, does this really always suffice? Even for free-boundary conditions? To resolve these issues, this talk examines the consequences of the violation of Ericksen's inequality $K_{22} \geq K_{24}$ to ascertain whether they are all harmless. The ultimate conclusion is that Frank's free energy is not apt to describe the elasticity of CLCs because it entails paradoxes that arise if above Ericksen's inequality is violated. This is not only a destructive talk; it also proposes an amendment to the Frank's theory, in the form of a quartic correction to the free-energy density, which promises to reinterpret correctly all experimental findings without leading to any paradox.
This is a joint work with Epifanio G. Virga.

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DYNAMIC IMAGINATION FOR SOLVING GEOMETRIC PROBLEMS

DANIELE PASQUAZI

In this study we were interested in investigating the ability of preadolescents (12-14 years old) to find effective solution strategies in geometric problems in which the figures were already provided with the text. The topic of the problems was surface equivalence. An initial test revealed that not even half of the students answered the items correctly and that the proposed solution strategies were, in most cases, rather convoluted. We considered this to be due to the fact that the students remained too stuck studying the initial figure of the problem, which otherwise, by imagining simple movements, could be transformed into an equivalent one for which it was easier to find the area.

We believe that by working on geometric perception we can develop this dynamic imagination. Since there are no specific studies on the encoding of geometric figures, we have referred to models for word encoding [2] that have also been experimentally verified [6]and to encoding processes during the observation of an object [5]. We know that the power of such neuronal activations depends on the habit we have of seeing an image.

We also wanted to consider the neuronal activations following interaction with mathematical manipulatives. It is known from studies on primates [4], and from studies on humans [1], that we grasp an object in the way that is most effective for our purposes. The neuronal circuits involved are not only activated when the individual's hand interacts with the object but also when he/she simply remembers the interaction with it [3].

The students involved in the study then performed specific treatments. In order to compare the effectiveness of these treatments, the students were divided into two groups although the objectives of the assigned tasks were the same. The difference between the two treatments was that, in order to formulate and test their hypotheses, the students in one group could use traditional tools while those in the other group could use manipulatives representing geometric figures and designed taking into account the neurophysiological aspects mentioned above.

After the treatments, all students were tested again. We found that most of the students in both groups were clearly able to identify the known figure against which to transform the initial figure using appropriate movements. However, about 6 months later, with another test, we found that the students who had worked with traditional methods showed no further improvement in contrast to the students in the other group. We concluded that interactions with manipulatives help to recover related mathematical facts even after a long time.

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ATTRACTORS FOR A FLUID-STRUCTURE INTERACTION PROBLEM WITH TIME-DEPENDENT PHASE SPACE

CLARA PATRIARCA

This talk will be concerned with the longterm dynamics of a fluid-structure interaction problem describing a Poiseuille inflow through a 2D channel containing a rectangular obstacle. Physically, this models the interaction between the wind and the deck of a bridge in a wind tunnel experiment, as time goes to infinity. Due to the nature of the interaction problem, the fluid domain depends on time in an unknown fashion. As a result, the solution operator associated to the system acts on a time-dependent phase space, and it cannot be described in terms of a semigroup nor of a process. Nonetheless, we are able to extend the notion of global attractor to this particular setting, and prove its existence and regularity. This provides a strong characterization of the asymptotic behavior of the problem. Moreover, when the inflow is sufficiently small, the attractor reduces to the unique stationary solution of the system, corresponding to a perfectly symmetric configuration. See also [1].

This is a joint work with Filippo Gazzola, Vittorino Pata.

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DERIVED EQUIVALENCES FOR COMMUTATIVE NOETHERIAN RINGS

SERGIO PAVON

This is joint work with J. Vitória [1].

Let R be a commutative noetherian ring. The bounded derived category $\mathsf{D}^b(\mathsf{mod}(R))$ of the abelian category of finitely presented R-modules $\mathsf{mod}(R)$ is a triangulated category which contains $\mathsf{mod}(R)$. It also contains many other abelian categories: the *hearts of t-structures*, which in turn have their own bounded derived category. The goal of this talk is the following:

Theorem 1. *The hearts of intermediate t-structures of $\mathsf{D}^b(\mathsf{mod}(R))$ all have equivalent bounded derived category.*

This fact is useful, since abelian categories with equivalent derived categories share some invariants; but it is also a bit surprising, since the analogue for non-commutative noetherian rings (say, finite dimensional k-algebras) is completely false (as it would imply that every silting complex is tilting, making Silting Theory pointless).

We will trace the steps of the proof, which involves looking at the unbounded derived category $\mathsf{D}(\mathsf{Mod}(R))$, lifting our intermediate t-structures to certain compactly generated intermediate t-structures of $\mathsf{D}(\mathsf{Mod}(R))$, and showing that they can be obtained from the standard one by successive "deformations" (HRS-tilts), each of which does not change the derived category.

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RESULTS ABOUT THE CHOW RING OF MODULI OF STABLE CURVES OF GENUS THREE

MICHELE PERNICE

In this talk, we will discuss some results concerning the Chow ring of $\overline{\mathcal{M}}_3$, the moduli stack of stable curves of genus three.

We will start by describing the state of the art regarding the intersection theory of moduli stacks of (stable) curves, listing some important results and computations in the field.

Then, we will briefly introduce the notion of the equivariant intersection theory, listing the property we need and explaining some example. The reference for this part is [1].

Subsequently, we will discuss the strategy we want to apply to compute the Chow ring of the moduli stack of $\overline{\mathcal{M}}_3$, namely introducing the so-called "patching lemma" and explaining what are the issues with it and how to solve them. In doing so, we will apply the strategy for the case of $\overline{\mathcal{M}}_2$ to better understand the potential (and also the limitations) of this strategy. The same strategy was used in [2] to compute the integral Chow ring of $\overline{\mathcal{M}}_{2,1}$.

Finally, we will describe how the strategy applies in the case of $\overline{\mathcal{M}}_3$ and explain what issues have to be solved in order to conclude the computation.

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GAUSS–NEWTON TYPE METHOD FOR SOLVING NONLINEAR LEAST-SQUARES PROBLEMS

FEDERICA PES

Nonlinear least-squares problems have applications in many fields of science and engineering, for example, if a physical system is modeled by a nonlinear function, the unknown parameters can be estimated by fitting experimental observations by a least-squares approach. The Gauss–Newton method is a classical algorithm typically used for solving nonlinear least-squares problems. In this work, we present a Gauss–Newton type method for the computation of the minimal-norm solution, which relies on two relaxation parameters to ensure convergence. In particular, we show that the iteration of *minimal-norm Gauss–Newton method* is obtained from that of Gauss–Newton by adding a correction vector. To ensure convergence it is fundamental to control the step length for the Gauss–Newton iteration, as well as to control the step length for this correction term. When the nonlinear function is ill-conditioned, we consider some regularization techniques for the solution. Numerical experiments are presented to illustrate the performance of the algorithm. This is a joint work with Giuseppe Rodriguez [1, 2].

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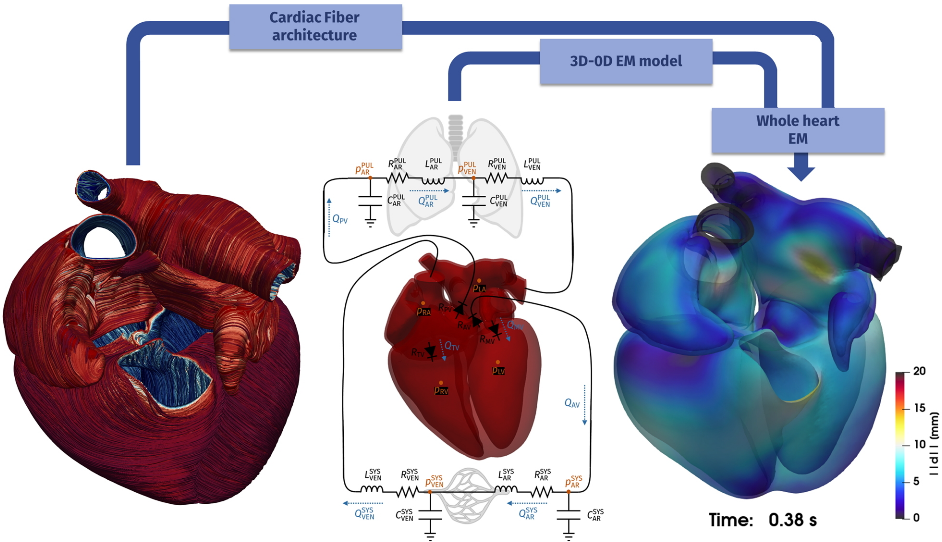
MATHEMATICAL AND NUMERICAL MODELING OF CARDIAC FIBER GENERATION AND ELECTROMECHANICAL FUNCTION: TOWARDS A REALISTIC SIMULATION OF THE WHOLE HEART

ROBERTO PIERSANTI



(a) (b) (c)

FIGURE 1. Graphical abstract: a) cardiac muscular fiber architecture; b) 3D-0D whole heart model; c) full heart electromechanical simulation.

Even though some area of heart modeling reached a certain level of maturity, whole heart models are a far reaching endeavour and are still in their infancy. This work provides a detailed fully coupled multiscale mathematical and numerical model of cardiac electromechanics (EM) of the whole human heart. Two crucial factors for accurate numerical simulations of cardiac EM are: reconstructing the muscular fiber architecture; accounting for the interaction between the heart and the circulatory system. With the aim of facing the challenges formerly described, the main contributions in this work move along two strands. On the one hand, we developed a unified mathematical framework to prescribe myocardial fibers orientation in computational four chamber heart geometries, see Figure 1(a) [1]. On the other hand, we provide a biophysically detailed cardiac 3D EM model coupled with a 0D closed-loop lumped parameters model of the whole circulatory system, see Figure 1(b) [2]. The validity of the whole heart 3D-0D model was demonstrated through EM simulations with physiological activation sites in realistic cardiac computational domain, see Figure 1(c).

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AUTOMORPHISMS ON ALGEBRAIC VARIETIES: K3 SURFACES, HYPERKÄHLER MANIFOLDS, AND APPLICATIONS ON ULRICH BUNDLES

YULIETH PRIETO–MONTANEZ

One of the main tools to study the geometry of complex algebraic varieties is the group of automorphisms. The goal of this talk is first of all to show how a non–empty automorphism group can determine naturally the geometry of some hyperkähler manifolds *or* can characterize varieties admitting objects (e.g., vector bundles) with specific conditions. It is not our purpose to determine the group of automorphisms of some algebraic varieties, instead, we use the action of (birational maps) automorphisms in different contexts, to restrict the geometry of the variety or in some cases the existence of certain objects.

As time is pressing, I will present only some ideas of the following results:

Theorem 1. *Let X be a smooth projective variety with non–empty $Bir(X)$.*

(1) *If X is a K3 surface admitting a symplectic automorphism σ of order three, then $\rho(X) \geq 13$ and if $\rho(X) = 13$ the Nerón–Severi group of X determines uniquely the K3 surface of the minimal resolution of $X/\langle\sigma\rangle$ and viceversa.*
(2) *If X is a hyperkähler manifold of $K3^{[n]}$–type admitting a symplectic birational map with non–trivial action on the discriminant group of $H^2(X, \mathbb{Z})$, then X is birational to a moduli space of (twisted) sheaves on a K3 surface.*
(3) *If the tangent bundle of X is an Ulrich bundle, the X is a rational homogeneous space with a rather large automorphism group. Moreover, if $\dim X = 1$, then X is the twisted cubic in $\mathbb{P}^3$ and if $\dim X = 2$, then X is the Veronese surface surface in $\mathbb{P}^5$.*

These are the main results on my PhD thesis [1]. Part (1) is a joint work with A. Garbagnati in [2]. Part (3) is a joint work with P. Montero, S. Troncoso, and V. Benedetti in [3].

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NONLOCAL NEUMANN BOUNDARY CONDITIONS: PROPERTIES AND PROBLEMS

EDOARDO PROIETTI LIPPI

We present some properties and existence results for problems involving the fractional p-Laplacian and a generic source term with nonlocal Neumann conditions on the boundary. Depending on the conditions on the source term, we use different techniques to obtain existence of solutions. When $p = 2$, we give a generalization considering problems involving both nonlocal and local interactions.

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THE STATIONARY DISTRIBUTION OF RANDOM WALKS ON RANDOM DIRECTED GRAPHS

MATTEO QUATTROPANI

The analysis of stochastic processes on random undirected graph is definitely one of the mainstream topics in modern probability. Nevertheless, the case in which the underlying graph is directed is much less studied and understood. One of the first difficulties one has to face when dealing with such geometries concerns the stationary distribution of the simplest stochastic process that can take place on a network: the simple random walk. In fact, in the case of undirected graphs, the stationary distribution coincides (up to normalization) with the degree sequence. Contrarily, in the case of directed graphs the stationary distribution depends on the features of the network on a complex non-local way.

During the talk I will present some recent finding on the stationary distribution of the random walk on the so-called *Directed Configuration Model*—a natural model of sparse random directed graphs. In particular, I will show some results on the extremal values of the stationary distribution and on the whole right-tail of its empirical distribution. In linear algebraic terms, this amount to find the right scaling of the largest entries of the Perron-Froboenius eigenvector of a random non-Hermitian stochastic matrix.

In the last part of the talk I will motivate the interest in such results by explaining how they can be turned into precise asymptotic estimates of other quantities related to the random walk behavior, such as mixing, hitting and cover times.

The result presented in the talk have been obtained jointly with my advisor Pietro Caputo and with Xing Shi Cai and Guillem Perarnau.

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TOPOSES OF MONOID ACTIONS

MORGAN ROGERS

In our thesis, which is based on [1], [2], [3] and [4], we studied toposes of actions of monoids on sets. We began with ordinary actions, producing a class of presheaf toposes which we characterized. As groundwork for considering topological monoids, we branched out into a study of supercompactly generated toposes (a class strictly larger than presheaf toposes). This enabled us to efficiently study and characterize toposes of continuous actions of topological monoids on sets, where the latter were viewed as discrete spaces. Finally, we refined this characterization into necessary and sufficient conditions for a supercompactly generated topos to be equivalent to such a topos of actions of a topological monoid.

In our talk we will explain the notion of topos and how we obtain special cases of toposes from monoid actions, before giving an overview of the strategy and main results of the thesis, including.

We will also present some of the ways that this work is being extended beyond actions on sets and some potential applications.

Several parts come from joint work with Jens Hemelaer.

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EXTREMALS AND CRITICAL POINTS OF THE SOBOLEV INEQUALITY

ALBERTO RONCORONI

The starting point of the talk is the sharp version of the classical Sobolev inequality in $\mathbb{R}^n$ proved in two independent papers: [8] and [1]. The Sobolev inequality has been object of several investigations and generalizations. In particular, in [5] the authors prove a Sobolev-type inequality in $\mathbb{R}^n$ for an anisotropic norm (i.e. a function $H : \mathbb{R}^n \to \mathbb{R}$ convex, positive 1-homogeneous and positive). The proof in [5] is based on the optimal transport technique and leads to the sharp anisotropic Sobolev inequality. In [4] we realize that the optimal transport technique can be used to prove a sharp anisotropic Sobolev-type inequality in convex cones of $\mathbb{R}^n$ (see also [6] and [2] for previous results).

Moreover, an important and well-studied result related to the Sobolev inequality is the classification of critical points, i.e. entire solutions to the so-called critical p-Laplace equation

$$\Delta_p u + u^{p^*-1} = 0 \quad \text{in } \mathbb{R}^n \,, \tag{1}$$

where Δ_p is the usual p-Laplace operator and p^* is the Sobolev critical exponent, explicitly

$$\Delta_p u := \mathrm{div}(|\nabla u|^{p-2}\nabla u) \quad \text{and} \quad p^* := \frac{np}{n-p} \,.$$

It has been shown (see e.g. [3, 7, 9]), exploiting the moving planes method, that positive solutions to (1) such that $u \in L^p(\mathbb{R}^n)$ and $\nabla u \in L^{p^*}(\mathbb{R}^n)$ can be completely classified. In the talk we will consider the anisotropic critical p-Laplace equation in convex cones of $\mathbb{R}^n$. Since the moving plane method strongly relies on the symmetries of the equation and of the domain, in [4] a different approach to this problem is introduced. In particular, this approach gives a complete classification of the solutions in an anisotropic setting. More precisely, we characterize solutions to the critical p-Laplace equation induced by a smooth norm inside any convex cone of $\mathbb{R}^n$.

The talk is based on the paper [4] in collaboration with G. Ciraolo and A. Figalli.

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MATHEMATICAL AND NUMERICAL MODELING OF CARDIAC ELECTROMECHANICS IN VENTRICLES WITH ISCHEMIC CARDIOMYOPATHY

MATTEO SALVADOR

Among the cellular-to-organ level physical phenomena contributing to the cardiac function, an important role is played by the coupling between the electrical activity of the heart and its mechanical contraction. For this reason, numerical simulations of ventricular electromechanics play nowadays a crucial role in computational cardiology and precision medicine. Indeed, it is of outmost importance to analyze and better address pathological conditions by means of anatomically accurate and biophysically detailed individualized computational models that embrace electrophysiology, mechanics and hemodynamics.

With this aim, we develop a novel electromechanical model for the human ventricles of patients affected by ischemic cardiomyopathy. This is made possible thanks to the introduction of a spatially heterogeneous coefficient that accounts for the presence of scars, grey zones and non-remodeled regions of the myocardium. We couple this 3D electromechanical model with 0D circulation models by an approach that is energy preserving. Our mathematical framework keeps into account the effects of mechano-electric feedbacks, which model how mechanical stimuli are transduced into electrical signals. Moreover, it permits to classify the hemodynamic nature of tachycardias. These aspects are very important for the clinical exploitation of our electromechanical model.

We propose two segregated-intergrid-staggered (SIS) numerical schemes to solve this 3D-0D coupled problem. Specifically, we consider two partitioned strategies for which different space-time resolutions are employed according to the specific core model. In particular, numerical models for cardiac electrophysiology require a finer representation of the computational domain and a smaller time step than those used for cardiac mechanics. For the first numerical scheme (SIS1), we introduce intergrid transfer operators based on Rescaled Localized Radial Basis Functions to accurately and efficiently exchange information among the several Partial Differential Equations (PDEs) of the electromechanical model. Different (potentially non-nested) meshes and first-order Finite Elements can be used for the space discretization of the PDEs. The second numerical scheme (SIS2) that we propose employs another flexible and scalable intergrid transfer operator, which allows to interpolate Finite Element functions between nested meshes and, possibly, among arbitrary Finite Element spaces for the different core models.

We also design a Machine Learning method to perform real-time numerical simulations of cardiac electromechanics. Our method allows to derive a reduced-order model (ROM), written as a system of Ordinary Differential Equations, in which the right-hand side is represented by an Artificial Neural Network (ANN), that possibly depends on a set of parameters associated with the model to be surrogated. This method is non-intrusive, as it only requires a collection of pressure and volume transients obtained from the full-order model (FOM). Once trained, the ANN-based ROM can be coupled with hemodynamic models for the blood circulation external to the heart, in the same manner as the original electromechanical model, but at a dramatically reduced computational cost. We demonstrate the effectiveness of the proposed strategy on two relevant contexts in cardiac modeling. We employ the ANN-based ROM to perform a global sensitivity analysis on both the electromechanical and the hemodynamic models. Then, we perform a Bayesian estimation of a couple of parameters starting from noisy measurements of two scalar outputs.

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NUMERICAL INVARIANT FOR MEASURABLE COCYCLES

FILIPPO SARTI

The theory of *numerical invariants* for representations can be generalized to measurable cocycles. This provides a natural notion of *maximality* for cocycles associated to complex hyperbolic lattices with values in groups of Hermitian type. Among maximal cocycles, the class of *Zariski dense* ones turns out to have a rigid behavior.

We will briefly introduce measurable cocycles and the machinery of numerical invariants, with a particular focus on their implementation via *boundary maps*. Then we will present a rigidity result about cocycles from complex hyperbolic lattices $\Gamma < \mathrm{pu}(1,n)$ into the Hermitian group $\mathrm{su}(p,q)$. In this context the *Toledo invariant* allows to introduce maximal cocycles. The combination of maximality, Zariski density and some ergodic theory implies *superrigidity* in the sense of Zimmer, which means that maximal Zariski dense cocycles $\Gamma \times X \to \mathrm{su}(p,q)$ comes from representations $\mathrm{pu}(1,n) \to \mathrm{su}(p,q)$ of the ambient group. Time permitting, we will see the crucial steps and the techniques involved in the proof. This is based on a joint project with Alessio Savini. [3, 2, 1]

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THE TOPOLOGY OF REAL ALGEBRAIC SETS WITH ISOLATED SINGULARITIES IS DETERMINED BY THE FIELD OF RATIONAL NUMBERS

ENRICO SAVI

The aim of this talk is to describe the topology of real algebraic sets by means of polynomial equations whose coefficients are as simple as possible. In [4] the authors provide an effective procedure to modify the coefficients of a given system of real polynomial equations getting a new system of polynomial equations whose coefficients are real algebraic numbers, while preserving the topology of the starting common solution set. However, when trying to get equations with rational coefficients their result only applies in few cases depending on the starting polynomial equations. Therefore, to investigate the open question about the possibility of describing over $\mathbb{Q}$ the topology of real algebraic sets, we introduce the notion of $\mathbb{Q}$-determined real algebraic set. In particular, $\mathbb{Q}$-determined nonsingular real algebraic sets are in some sense the minimal class of real algebraic sets, in terms of assumptions to be required, to develop new smooth approximation techniques over $\mathbb{Q}$ with regard to [3], [5] and [1]. Then, applying the mentioned new smooth approximation techniques over $\mathbb{Q}$, we get a relative version of the classical Nash-Tognoli's theorem over $\mathbb{Q}$ (see [2]), that is:

Theorem 1. *Every compact $\mathscr{C}^\infty$ manifold $M \subset \mathbb{R}^n$ containing $\mathscr{C}^\infty$ submanifolds M_i of codimension one in general position, for $i = 1, \dots, \ell$, can be arbitrarily $\mathscr{C}^\infty$ approximated by a $\mathbb{Q}$-determined projectively $\mathbb{Q}$-closed nonsingular algebraic set $M' \subset \mathbb{R}^m$, for some $m \geq n$, containing $\mathbb{Q}$-determined nonsingular algebraic subsets M'_i of codimension one in general position, for $i = 1, \dots, \ell$, such that each M'_i approximates M_i, for every $i = 1, \dots, \ell$.*

Moreover, after interpolation techniques, resolution of singularities, applications of the above Theorem 1 and blowing down operations over $\mathbb{Q}$ we are able to get results also in non-compact and singular cases. Indeed, our main result, which i will explain in deep, is the following:

Theorem 2. *Every real algebraic set $V \subset \mathbb{R}^n$ with isolated singularities is semi-algebraically homeomorphic to a $\mathbb{Q}$-determined real algebraic set $V' \subset \mathbb{R}^m$, with $m \geq n$. Furthermore, the homeomorphism $\phi : V \to V'$ we construct has the following additional properties:*

(i) *it preserves nonsingular points and restricts to a Nash diffeomorphism between the nonsingular loci,*
(ii) *it extends to a semi-algebraic homeomorphism from $\mathbb{R}^m$ to $\mathbb{R}^m$,*
(iii) *it is arbitrarily $\mathscr{C}^0$-close to the inclusion map $\iota : \mathbb{R}^n \hookrightarrow \mathbb{R}^m$ on compact subsets of V and arbitrarily $\mathscr{C}^\infty$-close to ι on compact subsets of* $\mathrm{Nonsing}(V)$*.*

This is a joint work in progress with Riccardo Ghiloni.

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DEEP LEARNING APPROXIMATION OF DIFFEOMORPHISMS VIA LINEAR-CONTROL SYSTEMS

ALESSANDRO SCAGLIOTTI

In this talk we will explore the interplay between Deep Learning and Control Theory. The starting point are the papers [2, 3], where it was independently observed that Residual Neural Networks (ResNets) can be naturally interpreted as discretization of continuous-time control systems. In this framework, it is possible to show that the expressivity of a ResNet is strictly related to controllability properties of the underlying control system. In this regards, in [1] the authors considered a linear-control system in $\mathbb{R}^n$ of the form

$$\dot{x}(t) = \sum_{i=1}^{k} F_i(x(t))u_i(t), \qquad t \in [0,1], \tag{1}$$

where $F_1, \ldots, F_k : \mathbb{R}^n \to \mathbb{R}^n$ are regular controlled vector fields, and $u_1, \ldots, u_k \in L^2([0,1], \mathbb{R})$ are the admissible controls. For every $u = (u_1, \ldots, u_k) \in L^2([0,1], \mathbb{R}^k)$ we can consider the diffeomorphism $\Phi_u : \mathbb{R}^n \to \mathbb{R}^n$ defined as the evaluation at the final instant of the flow induced by (1), i.e.,

$$\Phi_u(x_0) := x^u_{x_0}(1)$$

for every $x_0 \in \mathbb{R}^n$, where $x^u_{x_0} : [0,1] \to \mathbb{R}^n$ is the solution of (1) corresponding to the control $u = (u_1, \ldots, u_k)$ and to the initial datum $x^u_{x_0}(0) = x_0$. In [1], under mild hypotheses on the controlled vector fields $F_1, \ldots, F_k$, it was proved that, given a diffeomorphism $\Psi : \mathbb{R}^n \to \mathbb{R}^n$ isotopic to the identity and a compact set $K \subset \mathbb{R}^n$, for every $\varepsilon > 0$ there exists a control $u \in L^2([0,1], \mathbb{R}^k)$ such that

$$||\Psi - \Phi_u||_{C^0,K} \leq \varepsilon.$$

Starting from this theoretical result, in [4] we introduced an optimal control problem to model this approximation task. Indeed, following a data-driven approach, we imagined to observe the action of the diffeomorphism Ψ on an ensemble of training points $\{x_1, \ldots, x_N\}$ with $N \geq 1$. Therefore, we considered the following minimization problem on the space of admissible controls:

$$\frac{1}{N} \sum_{j=1}^{N} |\Psi(x_j) - \Phi_u(x_j)|_2^2 + \frac{\beta}{2} ||u||^2_{L^2} \to \min, \tag{2}$$

where $\beta > 0$ is a hyper-parameter that tunes the L^2-norm regularization. The discretization of (1) leads to a ResNet, and the numerical resolution of (2) can be seen as the training process for the ResNet just obtained. We explore two possible training strategies. The first one consists in projecting onto a finite-dimensional subspace of $L^2([0,1], \mathbb{R}^k)$ the gradient flow induced by the functional (2). The second one relies on an iterative algorithm based on the Pontryagin Maximum Principle. Finally, we provide an estimate for the generalization error by means of a Γ-convergence result.

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MATERIAL AND NON-MATERIAL HERITAGE OF THE ITALIAN SCHOOL OF ALGEBRAIC GEOMETRY (1880-1950)

ELENA SCALAMBRO

The aim of this talk is to provide a global vision over a long period of the Italian School of algebraic geometry – a social group of primary importance for the history of mathematics in Italy – through the adoption of the 'heritage' investigation lens. In a continuous dialogue with the past, through a significant scientific legacy, many modern achievements feed their roots in such cultural ground, which hence deserves to be considered more closely in the light of the 'process of patrimonialization'. This conception implies a dynamic vision of mathematical knowledge and leads to a synthesis of a plurality of elements, based on the reconstruction of a link with the past but also recognised today as an integral part of our mathematical culture. The heritage point of view encompasses material and immaterial aspects, providing a unified vision of the Italian geometric tradition. The 'material' lens of inquiry is deeply rooted in the concept of the 'material culture of science', which invites to shift the focus on material sources of scientific endeavours. In the case of the Italian School, it appears therefore relevant the study of institutional and private book heritages. As regards the first, the library assets (personal library and miscellany) of C. Segre, G. Fano and A. Terracini are analysed, revealing the existence of a common culture and a set of readings shared between the members of the School. Moving to the second aspect, it is worth considering the university libraries of the main Italian centres of research in algebraic geometry (Rome and Turin), delving into the different forms of use and management and their role as centres of production, preservation and transmission of mathematical knowledge. As regards the 'non-material' heritage – which includes the circulation of issues, problems, theorems and methods, the commonality of research themes and mathematical practices, and the phenomena of 'mathematical sociability' – at least three aspects have to be examined: research, teaching and divulgation. Taking into account also unpublished documents and manuscripts – through which the material and non-material dimensions intertwine – and exploiting the potentialities of digital humanities softwares to manage complex data networks, it is possible to shed a new light on the group of Italian algebraic geometers, made up not only by outstanding mathematicians, but conceived as a community of mathematical practice, a *communauté savant*, characterised by specific heritage, shared culture and traditions, epistemological and linguistic patterns.

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K-STABILITY AND THE HITCHIN-CSCK SYSTEM

CARLO SCARPA

One of the most important and fundamental results in complex geometry is the Hitchin-Kobayashi correspondence on vector bundles, which links the stability of a holomorphic vector bundle with the existence of Hermite-Einstein connections on the bundle. This correspondence can be seen as an infinite-dimensional analogue of the Kempf-Ness Theorem, which relates the existence of solutions to a moment map equation to GIT-stability. This point of view has inspired a great deal of research in complex geometry, as many important structures of compact complex manifolds can be interpreted as the existence of solutions of some (infinite-dimensional) moment map equation.

Of particular interest is the existence of Kähler metrics with special curvature properties on compact Kähler manifolds, such as Kähler-Einstein metrics, constant scalar curvature (cscK) or extremal Kähler metrics. Inspired by the case of Hermite-Einstein metrics, it has been conjectured that the existence of cscK metrics should be equivalent to an algebraic condition called K-stability. Our goal is to extend this parallel between the Hermite-Einstein and the cscK problems by studying, on the category of polarized varieties, a system of equations analogous to Hitchin's Higgs bundle equations on vector bundles.

The system is obtained from an infinite-dimensional hyperkähler reduction, which gives a set of moment map equations for a Kähler metric and infinitesimal deformation of the complex structure, which plays the role of a Higgs field in the category of Kähler manifolds. The system reduces to the cscK equation when the infinitesimal deformation vanishes. This approach is based on an explicit construction of hyperkähler metrics by Biquard and Gauduchon and generalizes a previous result by Donaldson on complex curves. We formulate a generalization of K-stability that should characterize the existence of solutions to this system, and we will sketch the proof of this conjecture for toric manifolds. Based on joint work with Jacopo Stoppa.

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FANO MANIFOLDS WITH LEFSCHETZ DEFECT 3

SAVERIO ANDREA SECCI

The *Lefschetz defect* δ_X is a numerical invariant associated to a smooth complex, Fano variety X, and it depends on the Picard number of prime divisors contained in X. Explicitly, consider the real vector space $\mathcal{N}_1(X)$ of real 1-cycles up to numerical equivalence. Its dimension is the Picard number ρ_X. Now, for any prime divisor D in X we define $\mathcal{N}_1(D,X)$ as the image of the pushforward $\iota_*\colon \mathcal{N}_1(D) \to \mathcal{N}_1(X)$ induced by the inclusion $\iota\colon D \hookrightarrow X$, that is the subvector space of $\mathcal{N}_1(X)$ generated by the numerical classes in X of curves in D. Finally,

$$\delta_X := \max\{\operatorname{codim} \mathcal{N}_1(D,X) | D \text{ a prime divisor in } X\}.$$

The main property of δ_X is that if $\delta_X \geq 4$, then X is isomorphic to a product $S \times T$, with S a del Pezzo surface of Picard number $\rho_S = \delta_X + 1$ [1, Th. 3.3].

In this talk we discuss a classification result for smooth Fano varieties with $\delta_X = 3$, which provides a generalisation of [2] to any dimension and Picard number: although X is not necessarily a product, it still has a very explicit description. That is, there exist a smooth Fano variety T of dimension $\dim T = \dim X - 2$ and Picard number $\rho_T = \rho_X - 4$, and a fibration $\sigma\colon X \to T$ such that the fibres are del Pezzo surfaces, and σ factorises as a $\mathbb{P}^2$-bundle over T and the blow-up along three pairwise disjoint smooth, irreducible, codimension 2 subvarieties, horizontal for the $\mathbb{P}^2$-bundle over T. We explicitly describe all possible $\mathbb{P}^2$-bundles and centres of the blow-up.

Moreover, we see some applications of the structure theorem: we describe the fibres, the relative contractions and the different factorisations of σ, and finally we describe in more details the case $\rho_X = 5$ and conclude the classification for $\dim X = 4$.

This talk is based on a joint work with C. Casagrande and E. A. Romano [3].

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GEOMETRIC MEASURE THEORY UNDER LOWER RICCI CURVATURE BOUNDS: A NON SMOOTH PERSPECTIVE

DANIELE SEMOLA

In Geometric Analysis there is a celebrated connection between minimal hypersurfaces and Ricci curvature. This interplay essentially boils down to the presence of Ricci in the second variation formula for the area and it is source of difficulties when the minimal objects obtained through Geometric Measure Theory (currents, varifolds, sets of finite perimeter) are not smooth, in high dimension.
The striking growth of the synthetic theory of lower Ricci curvature bounds in recent years led to a natural question: how many of the classical results linking Geometric Measure Theory with Ricci curvature hold when also the ambient space is not smooth?
In this talk I will review some of the most recent developments in this setting and illustrate how they can be employed to solve classical open problems in Geometric Analysis.

The talk is based on joint works in collaboration with Luigi Ambrosio, Gioacchino Antonelli, Elia Bruè, Andrea Mondino, Enrico Pasqualetto and Marco Pozzetta.

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A GEOMETRIC SINGULAR PERTURBATION APPROACH TO EPIDEMIC COMPARTMENTAL MODELS

MATTIA SENSI

We study fast-slow versions of the SIR, SIRS and SIRWS epidemiological models [1], and of the SIRS epidemiological model on homogeneous graphs [2], obtained through the application of the moment closure method. The multiple time scale behavior is introduced to account for large differences between some of the rates of the epidemiological pathways.

Our main purpose is to show that the fast-slow models, even though in nonstandard form, can be studied by means of Geometric Singular Perturbation Theory (GSPT).

In particular, without using Lyapunov's method, we are able to not only analyze the stability of the endemic equilibria of the SIR and SIRS models, but also to show that in the remaining models limit cycles arise.

We show that the proposed approach is particularly useful in more complicated (higher dimensional) models such as the SIRWS model and the SIRS on homogeneous graphs, for which we provide a detailed description of their dynamics by combining analytic and numerical techniques. In particular, for the latter we show that the model can give rise to periodic solutions, differently from the corresponding model based on homogeneous mixing.

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SAMPLING KANTOROVICH OPERATORS FOR THE SOLUTION OF CONCRETE REAL WORLD PROBLEMS

MARCO SERACINI

Sampling Kantorovich operators (SKO) have been introduced in the mathematical literature to extend the results of the classic and generalized sampling theorems to class of not necessarily continuous functions ([3, 2]). Starting from the one dimensional case, their expression has been consequently reformulated in multidimensional setting. Effective application models can be deduced from the mathematical formulation of the operators, particularly suitable to describe the practical sampling procedure. Thanks to this, the SKO have been successfully applied to images and to other bidimensional data elaborations for the solution of both medical as engineering specific problems. In particular, the open problem of the extraction of the pervious lumen boundaries in the aortic vessels, in presence of atheroma, has been made possible even without the introduction of contrast medium, nowdays representing the gold standard clinical procedure. Moreover, the characterization of the macular fundus for the prevention and the early diagnosis of the retinopathy, has been approached.
Furthermore, in the engineering contest, the quantification of the thermal and the acoustic bridges have been reformulated: in the first case, a more precise esteem of the energy losses of the buildings have been achieved ([1]); in the second case, the noise sources in complex environment has been localized in 3D. The application of such methods in reliable times has been made possible thanks to a numerical optimization of the reconstruction algorithm ([4]). A review of the results, achieved using the SKO procedure, is presented particularly focusing on the applications and on the connected concrete real world problems.
This is a joint work with prof. G. Vinti and the research group of the Department of Mathematics and Informatics of the University of Perugia.

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ON SPECIAL SUBVARIETIES OF THE TORELLI LOCUS

IRENE SPELTA

In this talk, we will discuss the "Coleman-Oort" conjecture. Such conjecture dates back to the '80s. Initially formulated by Coleman, then it was endorsed by Oort in the '90s and it is still open, although several, partial results, also very recent, have been obtained in support of it. A very good historical account is given in [6].

We work over the complex numbers and we look at the Torelli locus $\mathcal{T}_g$ inside the moduli space $\mathcal{A}_g$ of g-dimensional principally polarized abelian varieties. The conjecture says that for genus g large enough there do not exist special (or Shimura) positive dimensional subvarieties $S \subset \mathcal{A}_g$ which are generically contained in the Torelli locus. Special subvarieties are totally geodesic with respect to the locally symmetric ambient geometry and contain CM points ([7, 5]).

It is known that for $g \leq 9$ the conjecture does not hold. Indeed, there are examples of special subvarieties of $\mathcal{T}_g$ (see [1, 6, 2, 4, 3]). They were obtained as families of Jacobians of Galois coverings $C \to C' := C/G$ with fixed monodromy, genera, number of branch points under a certain (sufficient) numerical condition, which we denote by $(*)$. These examples are 30+6, having respectively $g' := g(C') = 0, 1$, as presented in [2], respectively in [4].

First, we will describe how strong is the condition $(*)$. Indeed we will show that it bounds g' and, using this bound, we will prove that the 6 examples of [4] are the unique families of Galois coverings of curves of positive genus satisfying $(*)$. Then we will see that these families admit two fibrations in totally geodesic subvarieties, countably many of which are special (see [3]). This yields infinitely many new examples of positive dimensional special subvarieties of $\mathcal{T}_2, \mathcal{T}_3, \mathcal{T}_4$.

Finally, we move to a recent result shown in [9]. We will prove that the families of dihedral covers described in [8] determine two special subvarieties of $\mathcal{T}_2, \mathcal{T}_3$. In this way, we will trace out the first two examples of special subvarieties generically contained in the Torelli locus without using the condition $(*)$.

This is based on joint work with A. Ghigi and P. Frediani [3] and on [9].

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A PULLBACK FUNCTOR FOR L^2-COHOMOLOGY

STEFANO SPESSATO

The main argument of this seminar is a result of the author in L^2-cohomology ([2], [3]). The definition of L^2-cohomology is very similar to the definition of de Rham cohomology. Indeed, given an oriented, possibly not compact, complete Riemannian manifold (M, g), the k-th L^2-cohomology group of (M, g) is

$$H_2^k(M, g) := \frac{ker(d^k)}{im(d^{k-1})}. \tag{1}$$

The operator $d^k : \Omega_2^k(M, g) \longrightarrow \Omega_2^{k+1}(M, g)$ is the exterior derivative operator of smooth forms defined on $\Omega_2^k(M, g)$, which is the space of smooth forms ω such that both ω and $d\omega$ are square integrable.

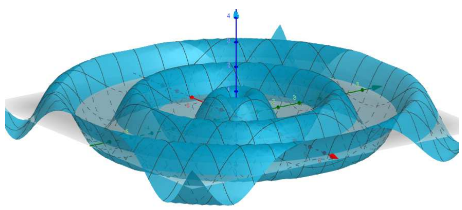
Differently from the de Rham cohomology, the pullback f^* along a map f is not well-defined as operator between $\Omega_2^*(N, h)$ and $\Omega_2^*(M, g)$. Several examples from [2] and [3] will be showed. During the seminar we will see how, fixed a map $f : (M, g) \longrightarrow (N, h)$, is possible to define a bounded operator $T_f : \Omega_2^*(N, h) \longrightarrow \Omega_2^*(M, g)$. The definition of T_f is inspired by a similar operator for compact manifolds defined by Hilsum and Skandalis in [1].

In order to define T_f we need two assumptions. The first one is about the Riemannian metrics: (M, g) and (N, h) have to be manifolds *of bounded geometry*, which means that there are an upper bound on the norm of the curvature and a lower bound on the injectivity radius. The second assumption is about the map f. We require that f is a *uniform* map, i.e. f is uniformly continuous and for each subset A of N, the diameter of $f^{-1}(A)$ is bounded in terms of the diameter of A itself.

As a consequence we obtain a contravariant functor

$$\begin{cases} \mathcal{T}(M, g) = H_2^k(M, g) \\ \mathcal{T}(f : (M, g) \longrightarrow (N, h)) = f^\circ : H_2^k(N, h) \longrightarrow H_2^k(M, g) \end{cases} \tag{2}$$

where $f^\circ[\omega] := [T_f \omega]$. Finally we obtain that the L^2-cohomology is a uniform homotopy invariant for manifolds of bounded geometry. Then, time permitting, we will see some consequences of this invariance.



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RANDOM DIFFERENTIAL TOPOLOGY

MICHELE STECCONI

In my Phd thesis [1] I developed some general methods to study the topological and geometric properties of random smooth maps. This topic is at the crossroad of Differential Topology and Probability.

In this talk, I will present such methods by studying the expected topology of singular sets of random (Kostlan) polynomials, when the degree grows to infinity [3].

This involves a generalization of the Kac-Rice formula [2], a probabilistic version of Thom transversality theorem [4], a Morse inequality for stratified manifolds [3] and a deterministic result on the behavior of Betti numbers under continuous perturbations [3].

This is a joint work with Antonio Lerario.

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A DISTRIBUTIONAL APPROACH TO FRACTIONAL SOBOLEV SPACES AND FRACTIONAL VARIATION

GIORGIO STEFANI

We develop a new distributional approach to fractional Sobolev spaces and to a newly-defined *fractional variation*. Given a parameter $\alpha \in (0,1)$, we consider the *fractional Riesz gradient* $\nabla^\alpha = \nabla I_{1-\alpha}$, where I_s is the Riesz potential operator of order s, and study the spaces $S^{\alpha,p}(\mathbb{R}^n) = \{f \in L^p(\mathbb{R}^n) : \nabla^\alpha f \in L^p(\mathbb{R}^n;\mathbb{R}^n)\}$, for $p \in [1,+\infty]$, and $BV^\alpha(\mathbb{R}^n) = \{f \in L^1(\mathbb{R}^n) : |D^\alpha f|(\mathbb{R}^n) < +\infty\}$, both defined in the usual distributional sense via the *fractional integration-by-parts formula* $\int_{\mathbb{R}^n} f \operatorname{div}^\alpha \varphi \, dx = -\int_{\mathbb{R}^n} \nabla^\alpha f \cdot \varphi \, dx$ valid for all functions $f \in C_c^\infty(\mathbb{R}^n)$ and vector fields $\varphi \in C_c^\infty(\mathbb{R}^n;\mathbb{R}^n)$. Our distributional approach allows to develop a quite rich and flexible theory, paralleling the known Sobolev–De Giorgi theory in this new fractional framework. We introduce new notions of *fractional Caccioppoli α-perimeter* and of *fractional reduced boundary* and prove several results, such as: a fractional version of De Giorgi's Blow-up Theorem [5]; the identification $S^{\alpha,p} = L^{\alpha,p}$ for $p \in (1,+\infty)$, where $L^{\alpha,p}$ is the usual *fractional Bessel potential space*, and an asymptotic study of the involved fractional operators, both in the pointwise and in the Γ-limit sense, via some new fractional interpolation inequalities [3, 6]; fine properties of BV^α-functions and of the fractional variation [4]; new fractional Leibniz and Gauss–Green formulas [7]. Very recently, our theory has revealed to be a promising field for the study of several new challenging applications of the theory of PDEs and of the Calculus of Variations to fractional operators [1, 2, 8, 9, 10, 11, 12, 13]. This is a research project in collaboration with Giovanni E. Comi. We acknowledge the collaboration of Elia Bruè, Mattia Calzi and Daniel Spector.

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ELLIPTIC LOOPS

DANIELE TAUFER

Given a local ring $(R, \mathfrak{m})$ with $6 \in R^*$ and an elliptic curve $E(R/\mathfrak{m})$, a new object called *elliptic loop* is defined as the set of points in $\mathbb{P}^2(R)$ lying over E with respect to the canonical projection. When the curve E has no even-torsion elements, the corresponding elliptic loop may be endowed with an operation inherited by the curve's addition law. This object is proved to be a power-associative abelian algebraic loop.

Although elliptic loops are not necessarily associative, they contain several abelian varieties defined over R by linear combinations of the curve Weierstrass polynomial and its Hessian. Those algebraic curves are called *layers*, as they provide a stratification of the affine part of their elliptic loop. The 0-layer coincides with the classical elliptic curve $E(R)$ lifting E.

Special properties are obtained when the underlying ring is $\mathbb{Z}/p^e\mathbb{Z}$, for which the infinity part of an elliptic loop may be realized as a direct product of two cyclic sub-loops. Moreover, the possible groups arising from layers over such rings are characterized by establishing a generator of their infinity parts.

When the underlying curve E has trace 1, the layer's group structure is employed for producing an isomorphism attack to the discrete logarithm on this family of curves. This attack has the same computational complexity as the known arithmetic approaches, but it involves only finite-precision objects.

Stronger properties are derived for small values of the exponent e, which lead to an explicit description of the infinity group and to characterizing the geometry of rational $|E|$-torsion points inside the overlying elliptic loop.

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HARMONIC ANALYSIS OF FEYNMAN PATH INTEGRALS

S. IVAN TRAPASSO

The Feynman path integral formulation of quantum mechanics is universally recognized as a milestone of modern theoretical physics. Roughly speaking, the core principle of this picture provides that the integral kernel of the time-evolution operator shall be expressed as a "sum over all possible histories of the system". This phrase entails a sort of integral on the infinite-dimensional space of suitable paths, to be interpreted in some sense as the limit of a finite-dimensional approximation procedure. In spite of the suggestive heuristic insight, the quest for a rigorous theory of Feynman path integrals is far from over, as evidenced by the wide variety of mathematical approaches developed over the last seventy years.

Among the several proposed frameworks, the closest one to Feynman's original intuition is probably the time-slicing approximation due to E. Nelson. In short, if $U(t)$ is the Schrödinger time evolution operator with Hamiltonian $H = H_0 + V$ (free particle plus a suitable potential perturbation), then the Trotter product formula holds for all $f \in L^2(\mathbb{R}^d)$:

$$U(t)f = e^{-\frac{i}{\hbar}t(H_0+V)}f = \lim_{n\to\infty} E_n(t)f, \quad E_n(t) = \left(e^{-\frac{i}{\hbar}\frac{t}{n}H_0}e^{-\frac{i}{\hbar}\frac{t}{n}V}\right)^n.$$

Integral representations for the approximate propagators $E_n(t)$ can be derived, so that the Trotter formula allows one to give a precise meaning to path integrals by means of a sequence of integral operators.

Notwithstanding the convergence results in suitable operator topologies, a closer inspection of Feynman's writings suggests that his original intuition underlay the much more difficult and widely open problem of the pointwise convergence of the integral kernels of the approximation operators $E_n(t)$ to that of $U(t)$. We recently addressed this problem by means of function spaces and techniques arising in the context of harmonic analysis. The toolkit of Gabor analysis has been fruitfully applied to the study of path integrals only in recent times, leading to promising outcomes.

With reference to the notation above, we consider a setting where H_0 is the Weyl quantization of a real quadratic form, hence covering fundamental examples such as the free particle or the harmonic oscillator. In addition, we introduce a bounded potential perturbation V whose regularity is characterized in terms of the decay in phase space of its windowed Fourier transform (such levels of regularity are encoded by the so-called modulation spaces).

We exploit techniques of Gabor analysis of pseudodifferential operators to prove that the problem of pointwise convergence has a positive answer under the previous assumptions. Precisely, we prove stronger convergence results which imply uniform convergence on compact subsets for the integral kernels in the Trotter formula.

Our results hold for any fixed value of $t \in \mathbb{R} \setminus \mathfrak{E}$, where $\mathfrak{E}$ is a discrete set of exceptional times - in that case the integral kernels are genuine distributions. Even in this case we are able to characterize the properties of such distribution kernels (they are mild distributions) and we derive weaker convergence results in the sense of modulation spaces.

We will also discuss the issue of rates of convergence for such results, obtained with a modification of the Trotter approximate propagators.

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BILINEAR CONTROL OF EVOLUTION EQUATIONS ON COMPACT NETWORKS

CRISTINA URBANI

In the recent work [1] we have solved the problem of exact controllabilty, in finite time $T > 0$, of the parabolic evolution problem

$$\begin{cases} u'(t) + Au(t) + p(t)Bu(t) = 0 \\ u(0) = u_0 \end{cases} \tag{1}$$

to some special target trajectories, called *eigensolutions*. Here p is a bilinear control. Denoted by $\{\lambda_k\}_{k\in\mathbb{N}^*}$ the eigenvalues of A and by $\{\varphi_k\}_{k\in\mathbb{N}^*}$ the associated eigenfunctions, the jth eigensolution of (1), $\psi_j(t) = e^{-\lambda_j t}\varphi_j$, is the solution of (1) for $p = 0$ and $u_0 = \varphi_j$.

The hypotheses to apply our result are linked to the null controllability of the following linearized problem

$$\begin{cases} u'(t) + Au(t) + p(t)B\varphi_j = 0 \\ u(0) = u_0 \end{cases}$$

and to the associated control cost. Sufficient conditions to have a suitable control cost, which allows to apply our controllability result, are an uniform gap condition of the eigenvalues of A and a lower bound for the Fourier coefficients of $B\varphi_j$. Observe that, because of the gap condition, the results of [1] are mostly applicable to low dimensional problems.

Therefore, it is reasonable to apply our controllability result to parabolic evolution equation on network structure, which are essentially one-dimensional domains. However, by considering the following dynamics

$$\begin{cases} u_t(t,x) - \Delta u(t,x) + p(t)Bu(t,x) = 0 \\ u(0,x) = u_0(x) \end{cases}$$

on a graph, one soon realizes that the eigenvalues of the Laplacian do not verify an uniform gap.

In [2] we adapted the controllabilty result of [1] to the case of a weaker gap condition of the eigenvalues. Thus, we were able to prove controllability to eigensolutions for the above problem on star and tadpole graphs.

This is a joint work with P. Cannarsa and A. Duca.

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A TOPOLOGICAL ANALYSIS OF COMPETITIVE ECONOMIES

NICCOLÒ URBINATI

Consider a multitude of decision makers whose choices are affected by the totality of others' actions, and yet negligible when considered individually. Whether agents act independently or coordinate their moves, the global outcome of their interaction is driven only by the aggregate result of their decisions. This situation is typical of competitive economies, where the aggregation process associates each group of agents with a set of bundles representing their mean demands.

In this presentation we ask what conditions ensure that aggregating many agents has a convexifying effect on their mean demands. The problem is an old one and characterizes competitive economies. We focus on the case of economies with an infinite-dimensional commodity space, where the classical results based on Aumanns' non-atomic representation of agents do not apply.

The question translates in the following mathematical problem: let $\mathcal{R}$ be a Boolean ring, E a locally convex linear space and $\varphi\colon \mathcal{R} \to 2^E$ a correspondence that is additive in the sense that $\varphi(0) = \{0\}$ and $\varphi(a \vee b) = \varphi(a) + \varphi(b)$ whenever a and b are disjoint. Under what conditions is the range $\bigcup\{\varphi(a) : a \in \mathcal{R}\}$ a convex and weakly compact set?

Our main results gives conditions under which an additive correspondence has a convex and weakly compact range. We base our approach on a topological reformulation of the "saturation property", a condition introduced in [1] and recently employed to prove specific extensions of Lyapunov's Theorem on the range of vector measures (see [2] and its references).

When included in the economic model, our conditions provide further insights on the nature of competitive economies in an infinite-dimensional context. We use our results to extend classical properties on the veto-power of small coalitions in the spirit of [3] and [4].

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SECOND ORDER HOMOGENEOUS HAMILTONIAN OPERATORS AND PROJECTIVE GEOMETRY

PIERANDREA VERGALLO

In 1983, B. A. Dubrovin and S. P. Novikov studied for the first time Hamiltonian structures for PDEs whose operator is homogeneous in the order of derivation. In particular, they introduced first order homogeneous Hamiltonian operators and, later, higher order operators of the same type. As a natural question, we wonder when a given system of PDEs possesses such a Hamiltonian structure. Necessary conditions for homogeneous quasilinear systems (also known as hydrodynamic type systems) to admit a Hamiltonian structure with homogeneous operators were found by different authors in the last years: for first order operators by S. Tsarev [1] and for third order operators by E. Ferapontov, M. Pavlov and R. Vitolo [2].

In this talk, we present similar conditions for second order homogeneous Hamiltonian operators as discussed in [3, 4]. These operators have the general form

$$P^{ij} = g^{ij}\partial_x^2 + b^{ij}_k u^k_x \partial_x + c^{ij}_k u^k_{xx} + c^{ij}_{kh} u^k_x u^h_x, \tag{1}$$

where $g^{ij}, b^{ij}_k, c^{ij}_k$ and c^{ij}_{kh} are functions depending on the field variables. Surprisly, the conditions found in this case can be solved, giving an explicit form for the conservative systems which admit second order structures.

The resulting systems and the operators turn out to be projective invariant and to possess interesting geometric properties:

Theorem 1. *Second-order homogeneous Hamiltonian operators are invariant under projective reciprocal transformations and the operators of dimension n can be put in bijection with 3-forms in the $n+1$-dimensional space $\mathbb{C}^{n+1}$.*

Finally, the previous result enables us to classify second-order homogeneous Hamiltonian operators in low dimensions ($n \leq 8$).

This is a joint work with Raffaele Vitolo.

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A FAMILY OF THREE-DIMENSIONAL VIRTUAL ELEMENTS FOR HELLINGER-REISSNER ELASTICITY PROBLEMS

MICHELE VISINONI

The Virtual Element Method (VEM) is a recent technology for the approximation of partial differential equation problems, which shares the same variational background of the Finite Element Method (FEM). The main features of this technique are its robustness to deal with general polygonal and polyhedral meshes, including hanging nodes and non-convex elements, and its flexibility to handle some interesting properties of the problem.

In this talk, we focus on the resolution of linear elasticity problems. More precisely, we consider the Hellinger-Reissner variational principle as the basis of our discretization procedure. In this framework it is well known that, for classical Galerkin schemes, designing an accurate method that preserves both the symmetry of the stress tensor and the continuity of the tractions at the inter-element is typically not a simple task. The fundamental reason behind this difficulty lies in the rigid structure of the polynomial approximation space. Therefore, our idea is to exploit the great flexibility of the VEM to avoid these troubles and design stable methods. Recently some Virtual Element schemes have been proposed and analyzed both for two and three-dimensional problems [1, 2, 3].

The aim is to present an extension to the three-dimensional case for a family of Virtual Element Methods. Some numerical tests are provided in order to show the validity and the potential of our analysis.

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CANONICAL SCALE SEPARATION IN 2D INCOMPRESSIBLE HYDRODYNAMICS

MILO VIVIANI

The fundamental rules governing a two-dimensional inviscid incompressible fluid are simple. Yet, to characterize the long-time behaviour is a knotty problem. The fluid's motion is described by Euler's equations: a non-linear Hamiltonian system with infinitely many conservation laws. In both experiments and numerical simulations, coherent vortex structures, or blobs, emerge after an initial stage. These formations dominate the large-scale dynamics, but small scales also persist. In his classical work, Kraichnan qualitatively describes a forward cascade of enstrophy into smaller scales and a backward cascade of energy into larger scales. Previous attempts to model Kraichnan's double cascade use filtering techniques that enforce separation from the outset.

In this talk, we show that Euler's equations posses an intrinsic, canonical splitting of the vorticity function. The splitting is remarkable in four ways:

- it is defined solely via the Poisson bracket and the Hamiltonian,
- it characterizes steady flows,
- without imposition it yields a separation of scales, enabling the dynamics behind Kraichnan's qualitative description,
- it accounts for the "broken line" in the power law for the energy spectrum (observed in both experiments and numerical simulations).

The splitting originates from Zeitlin's truncated model of Euler's equations in combination with a standard quantum-tool: the spectral decomposition of Hermitian matrices. In addition to theoretical insight, the scale separation dynamics could be used for stochastic model reduction, where small scales are modelled by a suitable multiplicative noise.

This is a joint work with prof. Klas Modin.

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N-FIBER-FULL MODULES

HONGMIAO YU

At the "CIME-CIRM Course on Recent Developments in Commutative Algebra" conference in 2019, Matteo Varbaro introduced the notion of "fiber-full modules" providing a new proof of the main result of [1]. The starting point of studing N-fiber-full modules is to find some possible generalizations of this concept.

Suppose that A is a Noetherian flat $K[t]$-algebra, M and N are finitely generated A-modules which are flat over $K[t]$, and all of A, M and N are graded $K[t]$-modules. Varbaro showed in his talk that if M is fiber-full, then $\operatorname{Ext}^i_A(M, A)$ is flat over $K[t]$ for all $i \in \mathbb{Z}$. We introduced the "N-fiber-full up to h" modules and we considered the following question: if M is N-fiber-full up to h as an A-module, can we obtain the flatness of some $\operatorname{Ext}^i_A(M, N)$? In the first part of this talk we will see that

MainTheorem. *Let h be an integer. M is N-fiber-full up to h as an A-module if and only if* $\operatorname{Ext}^i_A(M, N)$ *is flat over $K[t]$ for all $i \leq h - 1$.*

After that we will see some applications of this theorem. A main consequence is that the notion "N-fiber-full up to h" allows us to infer interesting results whenever the special fiber M/tM has "nice" properties after removing primary components of big height. For example, we will see the following Theorem:

Theorem. *Let S be the polynomial ring $K[X_1, \dots, X_n]$ over a field K, let $I \subseteq S$ be a homogeneous ideal. Fixed a monomial order on S, we denote by* $\operatorname{in}(I)$ *the initial ideal of I with respect to this monomial order. If I is such that* $\operatorname{in}(I)^{\mathrm{sat}}$ *is square-free, then*

$$\dim_K H^i_{\mathfrak{m}}(S/I)_j = \dim_K H^i_{\mathfrak{m}}(S/\mathrm{in})_j$$

for all $i \geq 2$ and for all $j \in \mathbb{Z}$.

Another interesting observation is: if S is $K[t]$-fiber-full, then the graded Betti numbers are preserved going from I to $\operatorname{in}(I)$.

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SPECTRAL SENSITIVITY ANALYSIS OF ELECTROMAGNETIC CAVITIES

MICHELE ZACCARON

The study of electromagnetic cavities has many real world applications, for example in designing cavity resonators or shielding structures for electronic circuits.

In this talk we consider the following eigenvalue problem arising from time-harmonic Maxwell's equations

$$
\begin{cases} \operatorname{curl}\operatorname{curl} u = \lambda\, \varepsilon\, u & \text{in } \Omega, \\ \operatorname{div} \varepsilon u = 0 & \text{in } \Omega, \\ \nu \times u = 0 & \text{on } \partial\Omega. \end{cases} \tag{1}
$$

Here Ω is a bounded domain in $\mathbb{R}^3$ and represents the electromagnetic cavity, while ν is the outer unit normal to Ω and ε denotes the electric permittivity of the material medium filling the cavity Ω. The boundary condition $\nu \times u = 0$ on $\partial\Omega$ is used in the case the boundary models perfectly conducting walls.

It is not difficult to see that the spectrum of problem (1) is discrete, consisting of non-negative eigenvalues $\{\lambda_j\}_{n\in\mathbb{N}}$ of finite multiplicity which can be arranged in an increasing, divergent sequence

$$0 \le \lambda_1 \le \lambda_2 \le \cdots \le \lambda_n \le \cdots \nearrow +\infty.$$

In this talk we present some results concerning the dependence of the eigenvalues λ with respect to the shape Ω of the cavity and the permittivity parameter ε. In particular, we provide Hadamard-type formulas for the derivatives of the eigenvalues as well as Rellich-Nagy-type theorems describing the bifurcation phenomena of multiple eigenvalues, and apply them to certain constrained optimization problems. We also discuss the spectral stability of problem (1) and give some hints on possible future works on these issues.

The results presented in the talk have been obtained in collaboration with Pier Domenico Lamberti [1, 2] and Paolo Luzzini [3].

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MATHEMATICAL AND NUMERICAL MODELS FOR THE FLUID DYNAMICS OF THE HUMAN HEART

ALBERTO ZINGARO

In silico simulations of the heart and blood circulation represent a valuable tool to analyze the cardiac function and to enhance the understanding of cardiovascular diseases. In this thesis, we introduce a Computational Fluid Dynamics (CFD) model for the numerical simulation of the heart hemodynamics in both physiological and pathological conditions, by accounting for all the physical processes that influence cardiac flows: moving domain and interaction with electromechanics, transitional-turbulent flows, cardiac valves and coupling with the external circulation. We propose a volume-based displacement model for the left atrium in physiological conditions and we simulate the atrial hemodynamics by considering both idealized and patient specific-geometries [1]. A lumped-parameter (0D) closed-loop circulation model serves as input to provide the CFD simulations with flowrates, pressures and atrial displacement. We further extend the computational model to account for atrial fibrillation in patient-specific CFD simulations. We investigate the effects of atrial fibrillation on the left atrium hemodynamics and we quantify an increasing blood stasis, especially in the left atrial appendage, where a dramatic washout reduction is observed [2]. The transitional blood flow regime is simulated by means of the Variational Multiscale - Large Eddy Simulation (VMS-LES) method, acting as both a stabilization and a turbulence model. We investigate the role of the VMS-LES method in transitional hemodynamic flows: if relatively coarse meshes are used, numerical results suggest that the additional stabilization terms introduced by the VMS-LES method allow to better predict transitional effects and cycle-to-cycle blood flow variations than the standard Streamline Upwind Petrov-Galerkin method [1]. We simulate the hemodynamics of the left heart and we integrate the electromechanical activity in the CFD model by employing a 3D ventricular electromechanical model. We propose a novel preprocessing procedure that combines the harmonic extension of the ventricular electromechanical displacement with the motion of the left atrium based on the 0D model. To better match the 3D CFD with the remaining blood circulation, we devise a coupled 3D-0D model made of the 3D CFD model of the left heart and the 0D circulation model: from a numerical point of view, we solve the coupled model by a segregated scheme, and we develop algorithms to solve the integrated system comprising fluid dynamics, displacement, valves and circulation models. Numerical simulations on a healthy left heart show that biomarkers and flow patterns are accurately reproduced when compared with in-vivo data [3]. We then expand our computational model to the hemodynamics of the four heart chambers finally bringing to an integrated multiscale CFD model of the whole human heart. This represents one of the few attempts in the literature to model the whole heart hemodynamics.

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Session K - Fisica Matematica

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Session L - Analisi Numerica

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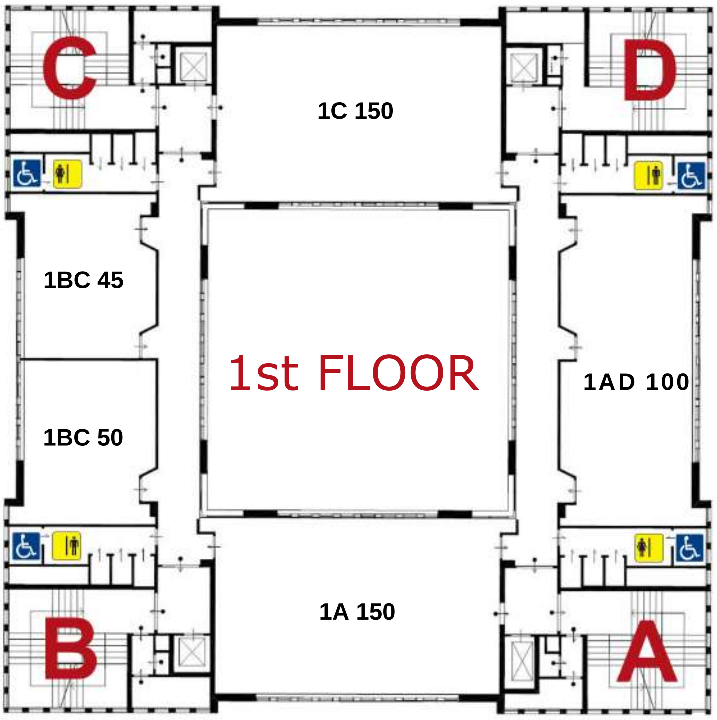

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